PSPI for High Performance Echo-Reconstructive Imaging

Ernesto Bonomi, Leesa Brieger, Carlo Nardone and Enrico Pieroni

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Centro di Ricerca, Sviluppo e Studi Superiori in Sardegna - Area Geofisica

Via Nazario Sauro 10, I-09123 Cagliari, Italy

Biographical notes

Ernesto Bonomi (ernesto@crs4.it) obtained his Ph.D. in Theoretical Physics at the University of Paris XI, Orsay and is currently the head of geophysics at CRS4. His earlier professional activities included work in statistical mechanics and computational physics; combinatorial optimization; geometrical modeling; and adaptive mesh generation.

Leesa Brieger (leesa@crs4.it) obtained a master's degree in Applied Mathematics at the University of Utah and is senior researcher in the geophysics group at CRS4. Her previous work has been in numerical methods and parallel computing for FE applications in material and environmental sciences, including development of parallel multigrid preconditioners for the conjugate gradient method.

Carlo Nardone (cmn@crs4.it) obtained his degree in Physics at the University of Rome "La Sapienza." Before being involved in the software development for the geophysics project at CRS4, he worked on the analysis of experimental signals on fusion devices at JET and elsewhere, and on the development of parallel codes for molecular dynamics and for geophysical flows.

Enrico Pieroni (ep@crs4.it) obtained his degree in Physics at the University of Pisa. He is currently involved in the parallel software and algorithm development for geophysics, acoustic inversion and wave modelling at CRS4. His earlier professional activities were in experimental high energy physics at SLAC and theoretical field theory at the University of Pisa.

Echo-reconstruction techniques for non-intrusive imaging have wide application, from subsurface and underwater imaging to medical and industrial diagnostics. The techniques are based on experiments in which a collection of short acoustic or electromagnetic impulses, emitted at the surface, illuminate a certain volume and are backscattered by inhomogeneities of the medium. The inhomogeneities act as reflecting surfaces or interfaces which cause signal echoing; the echoes are then recorded at the surface and processed through a "computational lens" defined by a propagation model to yield an image of the same inhomogeneities. The most sophisticated of these processing techniques involve simple acoustic imaging in seismic exploration, for which the huge data sets and stringent performance requirements make high performance computing essential.

Migration, based on the scalar wave equation, is the standard imaging technique for seismic applications [1]. In the migration process, the recorded pressure waves are used as initial conditions for a wave field governed by the scalar wave equation in an inhomogeneous medium. Any migration technique begins with an *a priori* estimate of the velocity field obtained from well logs and an empirical analysis of seismic traces. By interpreting migrated data, comparing the imaged interfaces with the discontinuities of the estimated velocity model, insufficiencies of the velocity field can be detected and the estimate improved [2], allowing the next migration step to image more accurately. The iterative process (turnaround) of correcting to a velocity model consistent with the migrated data can last several computing weeks, and is particularly crucial for imaging complex geological structures, including those which are interesting for hydrocarbon prospecting.

Subsurface depth imaging, being as it is the outcome of repeated steps of 3D seismic data migration, requires Gbytes of data which must be reduced, transformed, visualized and interpreted to obtain meaningful information. Severe performance requirements have led in the direction of high performance computing hardware and techniques. In addition, an enormous effort has historically gone into simplifying the migration model so as to reduce the cost of the operation while retaining the essential features of the wave propagation. The phase-shift-plus-interpolation (PSPI) algorithm can be an effective method for seismic migration using the "one-way" scalar wave equation; it is particularly well suited to data parallelism because of, among other things, its decoupling of the problem in the frequency domain.

The Exploding Reflector Model

The PSPI method will be discussed in the context of coincident source-receiver experiments. With the seismic data compression technique known as stacking, signals corresponding to all source-receiver pairs having the common midpoint $(x, y, 0)$ are collected into a single zero-offset trace which simulates a coincident source-receiver experiment. In such an experiment, the downward raypath and traveltime $t/2$ from the source to a point of reflection (reflector) is identical to the upward raypath and traveltime from the reflector to the receiver (see Fig.1). As a consequence, an equivalent trace would result from a source of appropriate intensity R initiated at the reflector and traversing the medium at half the original velocity. This is the so called exploding reflector model [3]. The field $R(x,y,z)$ of signal intensity which reproduces the ensemble of seismic traces (the seismic section) gives an acoustic image of the volume: large values of R correspond to sharp contrasts in the velocity field. Using the zero-offset seismic section $P(x,y,0,t)$ as a boundary condition and solving the scalar wave equation

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + \frac{\partial^2 P}{\partial z^2} - \frac{1}{v(x,y,z)^2}\frac{\partial^2 P}{\partial t^2} = 0 \tag{1}$$

in reverse time with zero initial conditions, the exploding reflector model allows us to interpret the migrated section $P(x,y,z,t=0)$ as a map of the local reflectivity, yielding an acoustic "picture" of the reflectors: $R(x,y,z) = P(x,y,z,t=0)$.

The Phase Shift Formula

The original phase shift migration method was formulated by J. Gazdag [4] as a fast and simple implementation of zero-offset data migration. Assume for the moment that v, the halved velocity of the medium, is constant. A way of solving Eq.(1) and thus arriving at our acoustic image $R(x,y,z) = P(x,y,z,0)$ is to use the depth z as the advancing variable along which to propagate the seismic section $P(x,y,0,t)$; this is known as depth extrapolation or continuation.

Eq.(1) written in the wavenumber-frequency domain (k_x, k_y, ω) is the second-order ordinary differential equation

$$\frac{d^2 \hat{P}(k_x, k_y, z, \omega)}{dz^2} = -k_z^2 \hat{P}(k_x, k_y, z, \omega) \; , \tag{2}$$

in which

$$k_z = \frac{\omega}{v}\sqrt{1 - \left(\frac{v}{\omega}\right)^2 \left(k_x^2 + k_y^2\right)} \; . \tag{3}$$

Eq.(2) has two characterisic solutions of the form

$$\hat{P}(k_x, k_y, z + \Delta z, \omega) = \hat{P}(k_x, k_y, z, \omega) e^{\pm i k_z \Delta z} \; , \tag{4}$$

relating the field at level z with that at level $z + \Delta z$ by a phase shift. Since agreement in sign between phase and frequency corresponds to a positive displacement of a wave in reverse time, and since by convention the z-axis points downward, for depth extrapolation we are interested only in the characteristic solution

$$\hat{P}(k_x, k_y, z + \Delta z, \omega) = \hat{P}(k_x, k_y, z, \omega) e^{i k_z \Delta z} \; , \tag{5}$$

which back-propagates in time. Remark that this is also a solution of

$$\frac{d\hat{P}}{dz}(k_x, k_y, z, \omega) = i k_z \hat{P}(k_x, , k_y, z, \omega) \; , \tag{6}$$

the paraxial or one-way wave equation, which provides a a hierarchy of migration methods [5] based on approximations of the one-way propagation in the space-frequency domain (x, y, ω).

If we use an inverse transform to map Eq.(5) to the space-frequency domain, we can then evaluate the field R and do the imaging of migrated data at level $z + \Delta z$ by noting that

$$P(x, y, z + \Delta z, t = 0) = \sum_{\omega} \hat{P}(x, y, z + \Delta z, \omega) \; . \tag{7}$$

Eqs.(3), (5) and (7) form the basis for the phase shift migration algorithm. They allow the exact inverse extrapolation of seismic data inside a homogeneous layer $]z, z + \Delta z]$ with constant velocity.

Since the power spectrum of the seismic source is band limited with a cut-off frequency far below the temporal Nyquist, mapping data into the space-frequency domain allows significant data compression. It is perhaps worth mentioning that to avoid data aliasing and thus improve the accuracy of the extrapolated wave, it is often preferable to substitute the ideal exponential operator with a "designed" one, to provide a highest attenuation between the passband region and the spatial Nyquist frequencies.

The phase shift formulation of migration leads to an elegant parallel implementation. In a pre-processing phase, the seismic traces $P(x, y, t)$ are transformed to the space-frequency domain (x, y, ω) by concurrent 1D-FFTs. $\hat{P}(x, y, \omega)$ is loaded from disk and distributed among processors by block of frequencies. All blocks are then concurrently transformed to the (k_x, k_y, ω) domain by 2D-FFTs. In addition, during downward continuation, three highly parallel steps are repeated for each value of z: (1) for each complex array, a parallel operator masks all spurious frequencies and shifts the phase of each entry, (2) partial sums over ω are concurrently performed, followed by the concurrent execution of 2-D FFTs, one per processor, (3) the imaging of the reflectors is done by summing over the transformed fields and then discharging onto disk the resulting migrated section $R(x, y, z) = P(x, y, z + \Delta z, t = 0)$.

PSPI: Phase Shift Plus Interpolation

Whereas seismic imaging in a stratified medium can be done with Eqs.(3), (5) and (7), the case with lateral velocity variations requires more attention. In this context the Fourier representation (2) of the scalar wave equation is meaningless and no straightforward representation of the solution as with the phase shift formula is possible. To overcome this difficulty and yet keep the computational complexity of the migration to a minimum, the wave propagation model is modified in order to construct a pure spectral method for downward continuation in an inhomogeneous medium.

The starting point is the phase shift formula (5), split into vertical and horizontal components and then modified to handle wave propagation inside the layer $]z, z + \Delta z]$ which has a laterally variable velocity field. The resulting first term governs vertically-travelling waves through the layer:

$$\hat{P}_0(x, y, z, \omega) = \hat{P}(x, y, z, \omega) e^{i\frac{\omega}{v}\Delta z} \ , \quad v(x, y, z) = v_z(x, y) \ . \tag{8}$$

The second term governs the horizontal correction for a reference velocity

$v_z^{(j)}$, one of $v_z^{(1)} < v_z^{(2)} < \cdots < v_z^{(n_z)}$:

$$\hat{P}^{(n)}(k_x, k_y, z + \Delta z, \omega) = \hat{P}_0(k_x, k_y, z, \omega) \exp\left(i(k_z^{(n)} - \frac{\omega}{v_z^{(n)}})\Delta z \right) , \tag{9}$$

with $k_z^{(n)}$, $n = 1, 2, \cdots, n_z$, given by Eq.(3) with velocity $v_z^{(n)}$.

The fields $\hat{P}^{(n)}(x, y, z + \Delta z, \omega)$ serve as reference data from which the final result is obtained by interpolation. With linear interpolation, the depth-continued wave field is given by

$$\begin{aligned} \hat{P}(x, y, z + \Delta z, \omega) &= \hat{P}^{(n)}(x, y, z + \Delta z, \omega) \frac{v_z^{(n+1)} - v_z(x, y)}{v_z^{(n+1)} - v_z^{(n)}} \\ &+ \hat{P}^{(n+1)}(x, y, z + \Delta z, \omega) \frac{v_z(x, y) - v_z^{(n)}}{v_z^{(n+1)} - v_z^{(n)}} , \end{aligned} \tag{10}$$

for all points (x, y, z) with $v_z^{(n)} \leq v_z(x, y) \leq v_z^{(n+1)}$. Finally, imaging of the reflectors at $z + \Delta z$ is obtained by the usual condition, Eq.(7). This is the essence of the PSPI method, as introduced by J. Gazdag and P. Sguazzero in [6] and outlined in the flowchart of Fig.(2).

The PSPI algorithm defined by Eqs.(8), (9) and (10) gives correct depth continuation for vertically-travelling plane waves and maintains high accuracy for small dips characterized by $\sqrt{k_x^2 + k_y^2} \ll |\omega|/v_z^{(n)}$. Although the introduction of laterally-variable velocities and the use of interpolating reference solutions do not have a well established mathematical basis, computer experiments show that this approach is in fact very reliable. PSPI is a practical alternative to other forms of migration – as long as the set of reference velocities is well-chosen (see below). Its implementational advantage is that each reference solution comes from a constant velocity extrapolation whose implementation inherits the parallel structure of the phase shift algorithm [7].

Optimal Reference Velocities

The velocities $v_z^{(n)}$ play a crucial role in PSPI migration of seismic data, but optimizing the choice of representative velocities has not been considered before. Now our group has used statistical arguments to optimally choose representative velocities for a given velocity field; the task is to highlight a minimal set of velocity values that predominate statistically in the propagation process. This can significantly diminish the computing cost of PSPI migration.

Consider a layer $]z, z+\Delta z]$ and the associated velocity field $v = v_z(x,y)$, discretized into $N+1$ equidistant values: $V_z^{(1)} = v_m < V_z^{(2)} < \cdots < V_z^{(N+1)} = v_M$ with v_m and v_M representing, respectively, the smallest and the largest values of the entire velocity field. Define as $P_z^{(k)}$ the probability that a given velocity present in layer z is contained in the kth velocity interval, $k = 1, \ldots, N+1$:

$$P_z^{(k)} = \text{prob}\left\{V_z^{(k)} \leq v < V_z^{(k+1)}\right\} \ , \qquad \sum_{k=1}^{N} P_z^{(k)} = 1 \ , \tag{11}$$

The resulting distribution represents a form of information about the structure of the velocity field. At each layer we would like to characterize the minimal number of velocities necessary to represent this information and then determine which velocities are most representative. The statistical dispersion of the distribution is measured by

$$S_z[P] = -\sum_{k=1}^{N} P_z^{(k)} \log P_z^{(k)} , \qquad 0 \leq S_z[P] \leq \log N . \tag{12}$$

This is nothing but the definition of the "statistical entropy" of the probability density distribution $P_z^{(k)}$. When there is no dispersion, namely when all velocities $v_z(x, y)$ for a given depth z are contained in a single velocity interval, then $S_z[P] = 0$; when the dispersion is maximal, namely all velocity intervals have the same statistical weight $1/N$, then $S_z[P] = \log N$.

For a given distribution over N intervals it is possible to condense the velocity intervals into $N_z \leq N$ new ones which have uniform statistical weight $1/N_z$ (maximal dispersion) and yet conserve $S_z[P]$; this is the minimal number of intervals necessary to represent the distribution. From the definition of the statistical dispersion, the number N_z of such intervals, rounded to the nearest integer, must be

$$N_z = \lfloor \exp(S_z[P]) + \frac{1}{2} \rfloor , \qquad 1 \leq N_z \leq N . \tag{13}$$

Now, in order to preserve $S_z[P]$, the reference velocities $v_z^{(j)}, j = 1, \ldots, N_z + 1$, which define the intervals of equal statistical weight must be such that

$$\frac{j}{N_z} = \int_{v_m}^{v_z^{(j+1)}} P_z(v) dv , \qquad j = 1, \ldots, N_z , \tag{14}$$

where $P_z(v) = P_z^{(k)}$ for $V_z^{(k)} \leq v < V_z^{(k+1)}$; $P_z(v)$ is the probability density distribution defined by 11. By defining

$$y_z^{(k)} = \int_{v_m}^{V_z^{(k)}} P_z(v) dv , \qquad k = 2, \ldots, N , \qquad y_z^{(1)} = 0, \quad y_z^{(N+1)} = 1 , \tag{15}$$

the computation of each reference velocity can be written as

$$v_z^{(j+1)} = V_z^{(k)} + \left[\frac{j}{N_z} - y_z^{(k)}\right] \frac{V_z^{(k+1)} - V_z^{(k)}}{y_z^{(k+1)} - y_z^{(k)}} \,, \qquad v_z^{(1)} = v_m \,, \qquad j = 1, \ldots, N_z \,, \tag{16}$$

where index k is such that: $y_z^{(k)} < j/N_z \leq y_z^{(k+1)}$.

Note that the constant velocity case, $v = v_z$, does not fit into the algorithm described, but also that the constant velocity case does not require the special treatment furnished by the algorithm.

We recognize in the above approach, Eqs.(13)–(16), a form of adaptive algorithm. The induced velocity grid $v_z^{(1)} < v_z^{(2)} < \cdots < v_z^{(n_z)}$, $n_z = N_z + 1$, has the intrinsic property of being fine around the maxima of $P_z(v)$ and coarse around the minima that we have identified. In geological situations where $v_z(x, y)$ is almost constant, only two or three reference velocities should be necessary to represent the information; in the presence of strong lateral fluctuations, more reference velocities are necessary. With this adaptive mechanism, accuracy must increase on average because of the statistical importance conferred to those velocities that contribute massively to the downward propagation of the wave field.

Reconstruction of a Complex Subsurface Model

We study a synthetic example, for which we know the velocity field to be correct, in order to examine the effect of PSPI simplifications in imaging a complex model.

Fig.3 illustrates the example velocity field for a vertical slice of a subsurface model: $\Delta x = \Delta y = \Delta z = 11.25$ m. The first layer represents the ocean, $v = 1500$ m/s, and the subsequent stratification represents a typical complex structure of the earth's crust; inside this region the velocity varies from 2500 to 5650 m/s. The nested structure is characterized by a strong lateral variation of the velocity field.

Fig.4 shows the resulting synthetic zero-offset seismic section (zero-offset traces simulated with $\Delta t = 4$ ms) obtained in two computational phases. In the first, many numerical seismic experiments are carried out, with different positions of simulated surface sources producing different illuminations of the subsurface model shown in Fig.3. Simulated back-propagating echoes are detected at $z = 0$. In the second phase, all simulated data are finally collected and stacked in the zero-offset representation.

For the seismic data extrapolation, the velocity field of Fig.3 was originally discretized into 30 levels for each value of z. Fig.5 displays the optimal number n_z of reference velocities as a function of z and the corresponding relative empiric frequency of n_z. The number of reference velocities required for the different layers in this example fluctuates from 1 to 13; on average only three velocities were necessary. Such an economy in the number of reference velocities required translates into an important reduction in computation time for depth extrapolation and PSPI migration.

The resulting migrated section, Fig.6, was computed using the PSPI algorithm. The complex structure of the subsurface model is largely reconstructed; information missing from the migrated section is also absent in the zero-offset seismic section and is not due to the approximations of PSPI. The arcs of hyperbola appearing at the bottom of Fig.6 are mostly due to scattering points present in the zero-offset section. Globally, the agreement between reflecting interfaces picked up by the PSPI migration and discontinuities of the velocity field is very good.

Acknowledgments

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Figure Captions

1. Common MidPoint stacking as an approximation of the zero-offset, exploding reflector model.

2. Flowchart of the PSPI algorithm.

3. 2D slice of a subsurface velocity model.

4. Simulated zero-offset time section relative to Fig.3. Polarity is mapped onto color, amplitude is mapped to saturation (i.e. white means no signal).

5. Velocity analysis for the model of Fig.3. Number of reference velocities n_z vs depth z and histogram of the frequency of n_z.

6. Depth-migrated seismic data relative to Figs.3 and 4.