The Angular Distribution of Asset Returns in Delay Space [*] [†]

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Abstract

We develop and apply a set of hypothesis tests with which to study changes in the angular distribution of points in delay space. Crack and Ledoit (1996) plotted daily stock returns against themselves with one day's lag. (This might be described as a plot in "delay space"). The graph shows these points collected along several rays from the origin. They correctly attribute this "compass rose" pattern to discreteness in the data. Asset prices move in discrete ticks. Our testing procedures allow one to test for changes in Crack and Ledoit's compass rose pattern. Our case study gives an example of such a change in distribution being caused by a change in regime. We plot the number of points along a given ray of the compass rose against the angle of that ray. This creates a "theta histogram" which describes the angular distribution of the points in delay space. We compare this distribution to a standard theta histogram created by a simple bootstrap procedure. The χ^2 test is then performed in order to estimate quantitatively the consistency of the actual data with the standard theta histogram. Extensions of this technique are discussed.

We apply our technique to an important episode of Russian monetary history. In the late nineteenth century, the "credit ruble" was a floating currency unlinked to precious metals. Generally, the finance ministry actively intervened to influence the ruble exchange rate. The one exception was during Nicolai Bunge's tenure as finance minister. Bunge's successor, Ivan Vyshnegradsky, was an unusually vigorous interventionist. The shift in regime from Bunge the non-interventionist to Vyshnegradsky the interventionist produced a marked change in the behavior of the ruble exchange rate. The angular distribution in delay space of the ruble's exchange rate against the German mark shifted dramatically under Vyshnegradsky. Hypothesis tests support the view that Vyshnegradsky's activism caused a disproportionate number of points of the compass rose to accumulate on the main diagonals in delay space. The theory of "Big Players" (Koppl and Yeager 1996) helps to explain why. Our results are consistent with those of Broussard and Koppl (1996) who use a GARCH(1,1) model.

I Introduction

Crack and Ledoit (1996) plot daily stock returns against themselves with one day's lag. Doing so produces the "compass rose" pattern of Figure 1. This pattern "is indisputably present in every stock". It "cannot be used for predictive purposes", however, because "it is an artefact of market microstructure" (p. 751). The existence of a non-zero tick size produces discreteness in the data which, in turn, generates the compass rose.

Crack and Ledoit view the compass rose as important for several reasons. First, it suggests the value of research into "the economic determinants of price discreteness" (p. 762). Their results also show that, because discreteness in the data produces a pattern, tests for various forms of autoregressive conditional heterskedasticity (ARCH) are likely to be biased as is the BDS test for chaos. Some standard tools of time-series analysis may be inapplicable to the discretized data of most asset markets.

Surprisingly, Crack and Ledoit do not call for new tools of time-series analysis specifically suited to the existence of ticks and of the compass rose pattern. Perhaps they doubt the possibility of applying algorithmic logic to the "subjective" compass rose pattern. Their explanation of the compass rose pattern uses "subjective language", they report, "because the above statement 'the compass rose appears clearly' is itself subjective" (p. 754). We show by example that new tools specifically suited to the compass rose and discretized data can be constructed. Our tools include hypothesis tests that are highly algorithmic (and thus "rigorous and objective"). We apply our techniques to a case study. We believe the results of our case support the view that our techniques are useful and worthy of further development.

II Hypothesis Testing and the Compass Rose Pattern

The compass rose was discovered by Crack and Ledoit (1996). They illustrate the compass rose with daily returns on Weyerhauser stock from December 6, 1963 to December 31, 1993. (See Figure 1.) Crack and Ledoit list three conditions for the compass rose pattern to emerge:

1. Daily price changes are small relative to the price level;

2. Daily price changes are in discrete jumps of a small number of ticks; and

3. The price varies over a relatively wide range.

The explanation of these three conditions is straightforward. Following their notation, let P_t and R_t be the price and return of some stock on day t. If price changes are small relative to price level ($(P_t - P_{t-1}) \ll P_t$), and ignoring dividends and splits, the following approximation holds:

$$R_{t+1}/R_t = \frac{(P_{t+1} - P_t)/P_t}{(P_t - P_{t-1})/P_{t-1}} \approx \frac{P_{t+1} - P_t}{P_t - P_{t-1}} = \frac{n_{t+1}h}{n_t h} = \frac{n_{t+1}}{n_t} \;, \qquad (1)$$

where h is the tick size and $n_t = (P_t - P_{t-1})/h$ is the day-t price change calculated in ticks. Equation (1) shows that the ordered pairs (R_t, R_{t+1}) will be close to the rays through the origin that pass through (n_t, n_{t+1}). If prices usually change by a small number of ticks, then most points will accumulate along the major directions of the compass rose.

Finally, Crack and Ledoit explain, if the price varied only slightly around the value P_t, a grid pattern would result, not the compass rose. "On any given ray (m, n) data points would cluster at discrete distances from the origin: $(mh/P_t, nh/P_t)$, $(2mh/P_t, 2nh/P_t)$, and so on" (754). Price variations produce "centrifugal smudging" which, in turn, produces the compass rose pattern.

Crack and Ledoit describe the compass rose pattern as "subjective". It is possible, however, to transform the data of the compass rose and apply analytically rigorous techniques to the transformed data. The transformation we propose might best be thought of as the result of a two step procedure. In the first step one expresses the points of the compass rose in polar coordinates. The point (R_t, R_{t+1}) is expressed as (r_t, θ_t) where

$$\begin{aligned} r_t &= \sqrt{R_t^2 + R_{t+1}^2} \\ \theta_t &= \begin{cases} arctan(R_{t+1}/R_t) & \text{if } R_t \geq 0 \\ arctan(R_{t+1}/R_t) + \pi & \text{if } R_t < 0, R_{t+1} \geq 0 \\ arctan(R_{t+1}/R_t) - \pi & \text{if } R_t, R_{t+1} < 0 \end{cases} \end{aligned} \qquad (2)$$

($arctan$ conventionally range from $-\pi/2$ to $\pi/2$).

The second step is to associate each θ_t, not with any of the corresponding r_t values, but with the number of such values corresponding to a narrow interval $\theta \pm \delta\theta$. We finally normalize by π in order to plot histograms in the interval $[-1, 1]$. The result may be called a "theta histogram". A theta histogram represents the angular distribution of asset returns in delay space. Figure 2 illustrates.

The theta histogram just described might be called the "empirical theta histogram". Before we can engage in hypothesis testing, we need a benchmark with which to compare it. We propose a simple bootstrapping procedure to create such a benchmark. To construct a bootstrapped theta histogram, one takes the observed relative frequency of each return in the data under study. Assume each period's return was draw from this distribution, and assume every period's return is independent of every other period. Repeated sampling from the empirical distribution of asset returns allows one to generate a bootstrapped theta histogram. Figure 3 gives an example. [1]

Hypothesis tests can be conducted by comparing the empirical and bootstrapped theta histograms. In the proposed tests described below, we test H_0, the null hypothesis that the $R(t)$ are independent. If the $R(t)/R(t-1)$ ratios are distributed in a sufficiently improbable manner (given H_0), then we infer some sort of dependence exists. We can say things such as "this theta has too many points" or "those thetas have too few points". The dependence we may infer is statistical dependence. It may be in the first moment, the second moment, or the 83^{rd} moment. It may be linear or nonlinear.

Let us consider just one narrow interval of theta, $\theta/\pi = \omega \pm \delta\omega$. Let n be the number of observations. That is, n is the number of $R(t)/R(t-1)$ ratios in our original data. Let p_ω be the fraction of points that should be on angle $\omega \pm \delta\omega$. If the $R(t)$ are independent, the average theta histogram will have the fraction p_ω of its point at $\omega \pm \delta\omega$. Thus with a sample of size n, the expected value for the number of points in the interval $\omega \pm \delta\omega$ is np_ω. Denote the observed value k_ω. We are now ready to build the quantity

$$\chi^2_{obs} \equiv \sum_\omega \frac{(k_\omega - np_\omega)^2}{np_\omega} \tag{3}$$

where $\sum_\omega k_\omega = n$. In the limit of a large number ν of ω partitions, such quantity has $P(\chi^2|\nu)$, the incomplete gamma function, as cumulative distribution (Press 1992). Selecting an arbitrary confidence level, say 0.05, as with any hypothesis test, one is able to reject H_0 if

$$P(\chi^2 \geq \chi^2_{obs}) \equiv Q(\chi^2|\nu) \equiv 1 - P(\chi^2|\nu) = \frac{1}{\Gamma(\chi^2_{obs})} \int_\nu^\infty e^{-t} t^{(\chi^2_{obs}-1)}\, dt \ < \ 0.05 \ , \tag{4}$$

where $\Gamma(x)$ is the gamma function.

The previous χ^2 test is meant to probe the empirical distribution as a whole. If some specific theta value is of interest because "has too many points", one can consider the interval $\omega \pm \delta\omega$

[1] It is clear that we can also construct empirical and bootstrapped versions of an "extensional histogram" analogous to the theta histogram. This graph would show the number of points at each distance from the origin. Studying in this way the "extensional distribution" of asset returns in delay space may enable us to address the issues of "fat tails" and extreme values.

and ask "What is the relative frequency of theta-histograms of size n in which $k_\omega \geq k_{obs}$"? We have a sequence of Bernoulli trials in each of which the probability of a hit is p_ω. For every k, with $0 \leq k \leq n$, there are $C(k,n) \equiv n!/[k!(n-k)!]$ ways in which you could get k hits. The probability of exactly k hits is $C(k,n)p_\omega^k(1-p_\omega)^{(n-k)}$. Thus, the probability $P(k_\omega \geq k_{obs})$ is

$$P(k_\omega \geq k_{obs}) \equiv Q(k_{obs}) = \sum_{k=k_{obs}}^{n} C(k,n)p_\omega^k(1-p_\omega)^{(n-k)} \ . \tag{5}$$

Again, H_0 is rejected if $P(k_\omega \geq k_{obs}) > 0.05$.

III An Application of our Technique

We apply our technique to an important episode in nineteenth-century Russian monetary history. [2] From the Crimean War of 1853-56 to 1897, Russian had a paper currency which floated against other currencies, including the Germany mark, a gold-standard currency. This was the period of the "credit ruble". During most of this period, the Russian finance ministry actively intervened in the foreign exchange market, hoping to influence the ruble's exchange rate. A notable exception was the period of Nicolai Bunge's tenure as finance minister. Bunge served from May 18, 1881 to January 13, 1887. He was a convinced and principled non-interventionist. Bunge's successor, Ivan Vyshnegradsky, was very different. He served from January 14, 1887 to September 11, 1892. (His stroke of April 7, 1892, however, put him largely out of the action). Vyshnegradsky was a highly active interventionist who meddled frequently in the Berlin market. Vyshnegradsky seemed to derive great pleasure from getting the better of the Berlin speculators in the ruble.

The contrast between Bunge and Vyshnegradsky is an unusually clear case of a change in regime from a simple policy rule of non-intervention to an activist, discretionary policy. It is thus a test case for the "Big Player" theory developed by Butos and Koppl 1993, Koppl 1996, Koppl and Langlois 1994, and Koppl and Yeager 1996. (See also Ahmed et al. n.d., Butos 1994, and Butos and Koppl 1995). According to this theory, the presence of a big, discretionary actor who is relatively insensitive to the discipline of profit and loss will induce herding in asset markets. Nicolai Bunge was not a Big Player because he maintained a

[2] The story we tell is related in Koppl and Yeager 1996, which we follow closely. They provide evidence for claims we make without support. The data we analyze was gathered by Yeager. It has been studied by Broussard and Koppl 1996, Koppl and Yeager 1996, and Yeager 1969 and 1984.

principled non-interventionist stance; he did not exercise his discretion. Vyshnegradsky did use his discretion and was thus a Big Player.

The root cause of the increased herding under Big Players is ignorance. By scrambling market signals, Big Players reduce the value of the market information little players use to make their choices. In this reduced state of knowledge, little players are more likely to study past price changes as a clue to the future course of prices. But that may create bandwagon effects. (See Figure 4.) Broussard and Koppl (1996) extend the point. They argue that GARCH effects are likely to be stronger under Big Players. "By reducing the value of all information", they explain, "Big Players increase the relative value of information about an asset's recent price behavior". An unusually large price change becomes the object of competing interpretations. "Some will see a trend, others will expect reversal. Whichever view happens to gain more adherents, the exaggerated attention paid to the price movement encourages another large movement" to follow. They fit a modified GARCH(1,1) model to the data and find an increase in GARCH effects under Vyshnegradsky. (See Figure 5). We use the same data to find a complementary pattern. Under Vyshnegradsky, a larger fraction of the points of the compass rose accumulated around the two main diagonals. Big price changes today produce big price changes tomorrow, though not always in the same direction.

Our data is constructed from a series created by Leland Yeager. Yeager used two contemporary German newspapers to find the ruble exchange rate in German marks per 100 rubles of bank notes. His data cover the period from January 02, 1883 to March 31, 1892. We created a return series from this data by taking the forward first differences of prices and normalising by the price (i.e. $R_i \equiv (P_i - P_{i-1})/P_{i-1}$). From this return series we calculated empirical and bootstrapped theta histograms as described above. The total number of samples in the bootstrapped histograms was 1000 times the length of the original series. The data points have been binned into 201 partitions with a resolution of $\theta/\pi = 0.01$. The probability associated with each bin is estimated to be $p_\omega = k/N$ with an error given by the corresponding bernoullian standard deviation $\sigma_{p_\omega} = \sqrt{p_\omega/N}$.

Figures 6 and 8 show the empirical theta histograms from the Bunge and Vyshnegradsky periods. Figures 7 and 9 show their corresponding bootstrapped theta histograms. For the Bunge period, the empirical and bootstrapped histograms are almost identical. For the Vyshnegradsky period, they differ. The asymmetry in the Vyshnegradsky period is evident.

Especially evident is the large number of points accumulated at $\theta/\pi = -0.5$. Days in which the ruble's exchange rate did not change tended to be followed by days in which its value fell. We don't know why this would be true.

For each period, we tested the hypothesis that the probability of a point at $\theta/\pi = -0.5$ is equal to the relative frequency of such points when returns are independent. The results are reported in Table 1.

For the Bunge period, the number of points n in the sample was 1224. If returns were independent, the probability of a point at $-0.5\,(\pm 0.005)$ would be $p_\omega = 0.0307$ and the expected value of the number of points would be $k_\omega = np_\omega = 37.6$. The actual number was $k_{obs} = 37$, well within the standard deviation for a bernoullian $(\sigma = \sqrt{np_\omega} = 6.1)$. We are therefore unwilling to reject the hypothesis of independence. In other words, for the Bunge period, under the assumption of independence, we do not have so improbably large or small number of points accumulating at -0.5 that we wish to reject the null hypothesis that returns are independent.

For the Vyshnegradsky period, the number of points in the sample was $n = 1581$. If returns were independent, the probability of a point at $-0.5\,(\pm 0.005)$ would be $p_\omega = 0.0244$ and the expected value of the number of points would be $k_\omega = 38.5$. The actual number was $k_{obs} = 50$. From equation (5) we calculate the probability to get such k or higher. This probability is $Q(k_{obs}) = 0.041$. Since $Q(k_{obs})$ is less than 0.05, our confidence level, we reject the hypothesis of independence. For the Vyshnegradsky period, under the assumption of independence, we have an improbably large number of points accumulating at -0.5. Our hypothesis test supports the conclusion one is likely to draw from looking at Figure 8: days in which the ruble's exchange rate did not change tended to be followed by days in which its value fell. (Unfortunately, it gives us no economic intuition about why).

The Big Players theory suggests we should find another difference between the Bunge and Vyshnegradsky periods. Under Vyshnegradsky, there should be a greater tendency for points to accumulate at $\theta/\pi = \pm 0.25$ and $\theta/\pi = \pm 0.75$. These are the values corresponding to the two main diagonals of the compass rose pattern. We expect the Big Player influence of Vyshnegradsky to encourage traders to pay more attention to price history, because all sources of information have been degraded by the Big Player's discretionary interventions. A large price change today will become the subject of interpretation in which some see a trend and other expect "correction". Whichever theory becomes more popular, a large-magnitude return today is likely to be followed by a return of similar magnitude, though not necessarily in the same direction.

Our confidence in this result is strengthened by an inspection of Figures 10 and 11. These figures show the compass rose pattern for the absolute value of returns. Since absolute values are non-negative, all points appear in the positive quadrant. Under Bunge, no ray is obviously accumulating too many or too few points. Note that the graph shows something close to a grid, with little centrifugal smudging. This is because the ruble exchange rate did not vary widely during Bunge's tenure as finance minister. (This is exactly the result predicted by Crack and Ledoit.) Under Vyshnegradsky, the 45-degree line seems to have collected more points than it would have if returns were independent. This difference between the Bunge and Vyshnegradsky periods is confirmed by hypothesis tests reported in Table 2.

In this case, in the Bunge period the compound probability for θ/π being within the intervals $\theta/\pi = \pm 0.25\,(\pm 0.005)$ and $\theta/\pi = \pm 0.75\,(\pm 0.005)$ is $p_\omega = 0.0610$, if returns were independent. Accordingly, the expected number of points is $k_\omega = 74.6$, while we observe $k_{obs} = 88$. From equation (5) we calculate the probability to get such k or higher. This probability is $Q(k_{obs}) = 0.065$. Since $Q(k_{obs})$ is more than 0.05, we are unwilling to reject the hypothesis of independence. For the Bunge period, under the assumption of independence, we do not have so improbably large or small number of points along the 45-degree line of Figure 10.

For the Vyshnegradsky period, $p_\omega = 0.0312$ and $k_\omega = 49.4$, while we observe $k_{obs} = 73$. Again from (5) we obtain $Q(k_{obs}) = 0.0008$, a value much smaller than our level of confidence. We can therefore reject the hypothesis of independence. For the Vyshnegradsky period, under the assumption of independence, we have an improbably large number of points accumulating at ± 0.25 and ± 0.75. Our hypothesis test supports the conclusion one is likely to draw from looking at Figure 11: large changes in the exchange rate on one day tend to be followed by similarly large changes the next day, though not necessarily in the same direction. This tendency is consistent with the theory of Big Players.

Finally, for each period, we tested the hypothesis of independence among returns using the χ^2 test. In order to avoid effects related to the number of points in the sample, we choose $n = 998$ for each series. A "mixed" series has been studied, taking the end of the Bunge period and the beginning of the Vyshnegradsky period of tenure, in equal proportions. The number of degrees of freedom ν coincides in our case with the number of bins, i.e. 201. Since ν is so large, we were able to use an asymptotic result to calculate the probability $P(\chi^2 > \chi^2_{obs}) \equiv Q(\chi^2_{obs}|\nu)$ (4). In fact, $Q(\chi^2_{obs}|\nu) \approx Q(x)$ where $x = \sqrt{2\chi^2_{obs}} - \sqrt{2\nu - 1}$ is a reduced variable normally distributed and $Q(x) \equiv 1/\sqrt{2\pi} \int_x^\infty \exp(-t^2/2)\, dt$ (Abramovitz and Stegun, 1972, formula 26.4.13).

The χ^2 test results are reported in Table 3. The test is in agreement with the previous tests on specific θ/π values. For the Bunge period $\chi^2_{obs} = 203.7$, very close to the expected value $\langle\chi^2\rangle = \nu = 201$. Therefore $Q(x)$ is quite large and well within the confidence level. For the Vyshnegradsky period $\chi^2_{obs} = 251.5$ and $Q(x) = 0.008 < 0.05$, therefore we can reject the hypothesis that Vyshnegradsky's empirical theta histogram is distributed as the corresponding i.i.d. returns histogram. This conclusion is even more probable for the "mixed" series, with $\chi^2_{obs} = 291.1$ and $Q(x) = 0.00002$.

IV Discussion

Building on Crack and Ledoit 1996, we have described and applied some new techniques of time-series analysis. We believe several considerations suggest that our techniques and others like them may be of fairly general interest in economics and finance.

Our techniques give us another way to look at GARCH phenomena. In the case studied in this paper, for example, we found evidence that GARCH effects may be due mostly to points accumulating along the two main diagonals of the compass rose. This gives us information on the dependence among returns that is not reflected in GARCH coefficients. Crack and Ledoit suggest that GARCH estimates may be biased by the same discreteness in time series data that produces the compass rose pattern. If this is true, then it may be desirable to have another technique of analysis capable of getting at volatility dynamics. We might even hope that pursuit of our techniques may eventually lead to progress in developing theoretical explanations for volatility dynamics.

Our techniques provide another engine for the discovery of new facts. In our case study, for example, we found that the Big Player influence of Vyshnegradsky produced a tendency for zero-return days to be followed by negative-return days. This discovery did not come from any statistical test, but from a graph, Figure 8. This method of discovery is the ancient technique of graphically based interocular trauma (GBIT). GBIT is often disparaged in econometrics classrooms, but it is useful for the discovery of facts. Mandebrot finds the distain of GBIT to have "become destructive" (p. 21). In his view, "the search for new concepts and conjectures are both helped by fine graphics. ... A formula can relate to only a small aspect of the relationship between model and reality, while the eye has enormous powers of integration and discrimination" (pp. 21-22). Of course, as Mandelbrot reminds us, "the eye sometimes sees spurious relationships" (p. 22). For this reason, we have attempted to devise hypothesis tests to complement GBIT. [3]

Our techniques are non-parametric. They require fewer maintained hypotheses. They allow us to avoid several assumptions that may sometimes be inappropriate. At least four questionable assumptions frequently made in the time-series analysis of economic data. These are *(i.)* the assumption of unbounded symmetric distributions, *(ii.)* the assumption of smooth price changes, *(iii.)* the assumption of ergodicity and *(iv.)* the assumption of linearity in intertemporal dependence. We discuss each in turn. We do not mean to deny that these assumptions are ever appropriate, only that they are always or necessarily so.

(i.) The assumption of unbounded symmetric distributions. An asset's price cannot fall below zero. It's return is bounded from below by -1. It is also bounded from above. The return cannot be so high, for instance that twice the total money stock would be required to pay it. Strickly speaking, returns cannot have normal or Levy-stable marginal distributions. Moreover, if the greatest lower bound to returns exceeds the least upper bound, the distribution will not be symmetric.

(ii.) The assumption of smooth price changes. Mandelbrot speaks against the assumption of continous price movements (1983, pp. 324-336). His comments, however, do not reveal

[3] Hayek's theory of mind (1952) provides an explanation for the fact that "we sometimes perceive patterns which we are unable to specify" (Hayek 1967, p.53). In this theory, all conscious thought presupposes "a system of rules which operate us but which we can neither state not form an image of and which we can merely evoke in others in so far as they already possess them" (Hayek 1967, p.62). (See Butos and Koppl 1993 and 1996 for expositions.) Hayek suggests a scale of complexity of patterns. Sufficiently complex patterns can be seen but not described. Somewhat less complex patterns, we may infer, can be described, but only after being seen. Sufficiently simple patterns may be discovered and described with fully articulated scientific methods. If all of this isabout right, then it would seem reasonable to encourage, not discourage, the use of graphical techniques.

an appreciation of the discretization imposed by a non-zero tick size. To assume a normal distribution of returns, a Levy-stable distribution, or any other continous distribution, is to assume that prices may move in any increment rather than by an integer number of ticks. Strickly speaking, this is a false assumption.

(iii.) The assumption of ergodicity. This assumption grows less common. Nevertheless, it is still commonly invoked. But much evidence points to persistence in the first and second moments of return series. (See Koppl and Yeager 1996 and Diebold and Lopez 1995). Our techniques look only for short-term dependence. But because they are nonparametric, they are not biased by the undetected presence of long memory.

(iv.) The assumption of linearity in intertemporal dependence. Typically the only kind of dependence considered among asset returns is linear dependence in the first and second moments. Our technique lets us test for statistical dependence without regard to the linear or nonlinear character of that dependence.

The use of the sort of non-parametric techniques we discuss in this paper allows us to aviod some of these potentially inappropriate assumptions. The use of such techniques also allows us to take advantage of falling costs of calculation. We can substitute computation for assumptions and derivations. (See Leijonhufvud 1993.)

Finally, techniques of the sort we have discussed are moments independent. By using tools such as the compass rose and theta histogram, we can examine our data without employing the potentially constraining categories of "first moment", "second moment", and so on. In our case study, this seems to have been a useful option. The accumulation of an improbably large number of points at $\theta/\pi = -0.5$ would be hard to describe in terms of moments.

We believe the techniques of time-series analysis we have discussed are likely to be useful in economics and finance. They may be developed in several directions and adapted to the special needs of different applications. It is our hope that other researchers will seek out and find innovative extensions and applications.

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Table captions

1: Hypothesis tests for $\theta/\pi = -0.5\,(\pm 0.005)$. In this and the following table, n is the number of points in the empirical distribution, p_ω is the probability that θ/π belongs to the interval considered according to the bootstrapped histogram, $k_\omega = np_\omega$, k_{obs} is the observed number of points in the interval considered, $\Delta k/\sigma = |k_{obs} - k_\omega|/\sigma$ is the normalized fluctuation observed, and finally $Q(k_{obs}) = \sum_{k=k_{obs}}^{n} P_k$ (where P_k is the Bernoulli probability of k observation in the bin) is the probability $P(k \geq k_{obs})$.

2: Hypothesis tests for $\theta/\pi = \pm 0.25\,(\pm 0.005)$ and $\pm 0.75\,(\pm 0.005)$.

3: χ^2 hypothesis tests. The empirical series include $n = 998$ observations histogrammed in $\nu = 201$ bins. $x = \sqrt{2\chi^2_{obs}} - \sqrt{2\nu - 1}$ is the reduced variable which is asymptotically distributed normally, $Q(x)$ is the corresponding probability $P(x' \geq x)$. The reduced variable x^* corresponding to the 0.05 confidence level, i.e. $Q(x^*) = 0.05$, is 1.645.

Figure captions

1: Crack and Ledoit's "compass rose" graph (scatter plot in delay space) for IBM daily stock returns, January 1, 1980 to October 8, 1992.

2: "Theta histogram" (angular distribution in delay space) of percent changes in the Russian Ruble's exchange rate against the German mark. The data used are daily prices from January 2, 1883 to March 31, 1892.

3: Bootstrapped theta histogram created using the same data used to construct the empirical theta histogram of Figure 2.

4: From Koppl and Yeager 1996 (ruble's exchange rates in German marks per 100 rubles).

5: From Broussard and Koppl 1996 (ruble's price percent changes).

6: Empirical theta histogram, Bunge period.

7: Bootstrapped theta histogram, Bunge period.

8: Empirical theta histogram, Vyshnegradsky period.

9: Bootstrapped theta histogram, Vyshnegradsky period.

10: Compass rose in absolute values, Bunge period.

11: Compass rose in absolute values, Vyshnegradsky period.

series	n	p_ω	k_ω	k_{obs}	$\Delta k/\sigma$	$Q(k_{obs})$
Bunge	1224	0.03072 (± 0.00016)	37.60 (± 0.20)	37	0.10	$>$ 0.5
Vyshnegradsky	1581	0.02436 (± 0.00012)	38.51 (± 0.19)	50	1.85	0.041

Table 1

series	n	p_ω	k_ω	k_{obs}	$\Delta k/\sigma$	$Q(k_{obs})$
Bunge	1224	0.06099 (± 0.00022)	74.66 (± 0.27)	88	1.54	0.065
Vyshnegradsky	1581	0.03123 (± 0.00014)	49.38 (± 0.22)	73	1.85	0.0008

Table 2

series	χ^2	x	$Q(x)$
Bunge	203.696	0.159	0.436
Vyshnegradsky	251.501	2.403	0.0082
mixed	291.102	4.104	0.00002

Table 3

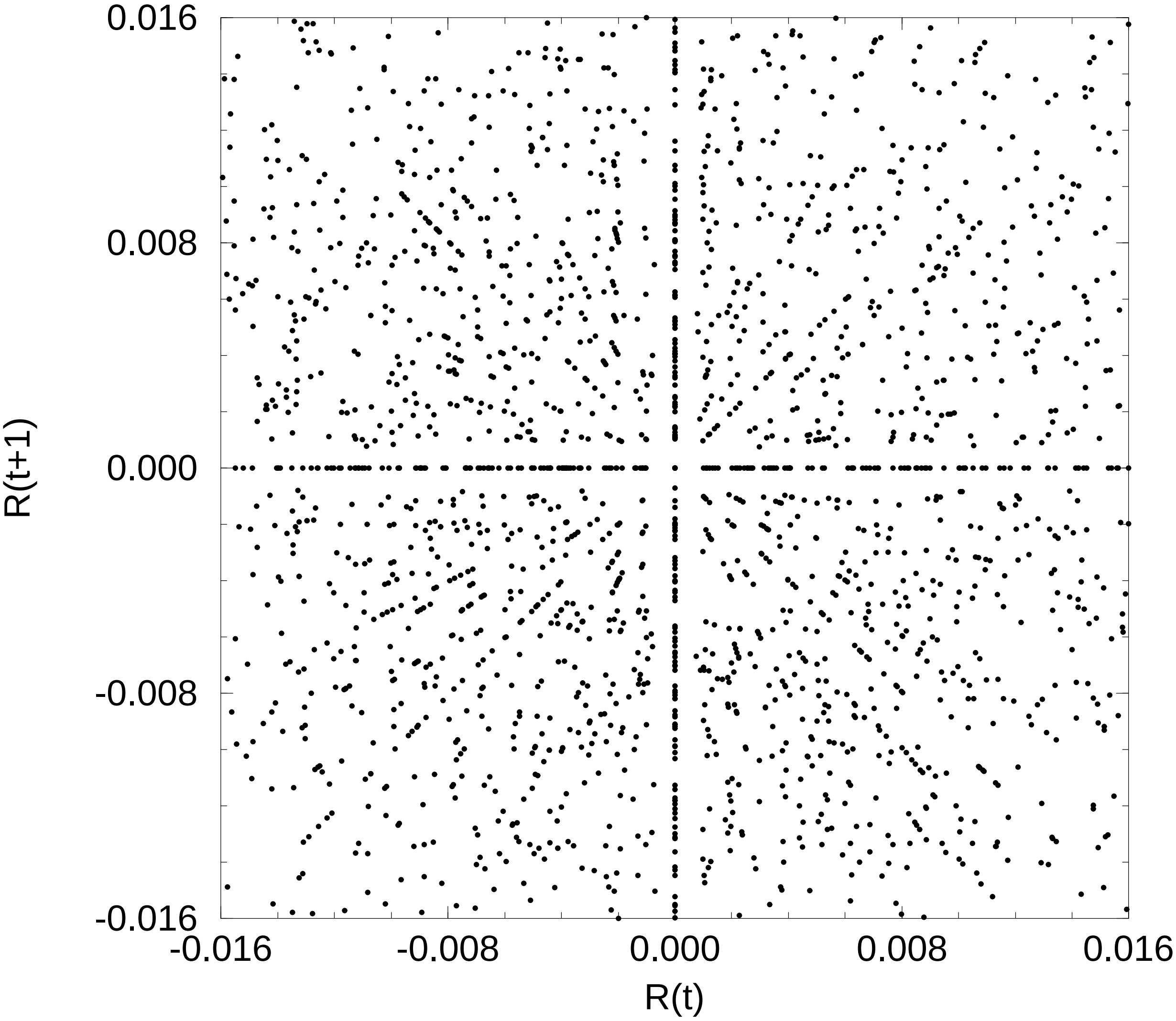


Fig.1

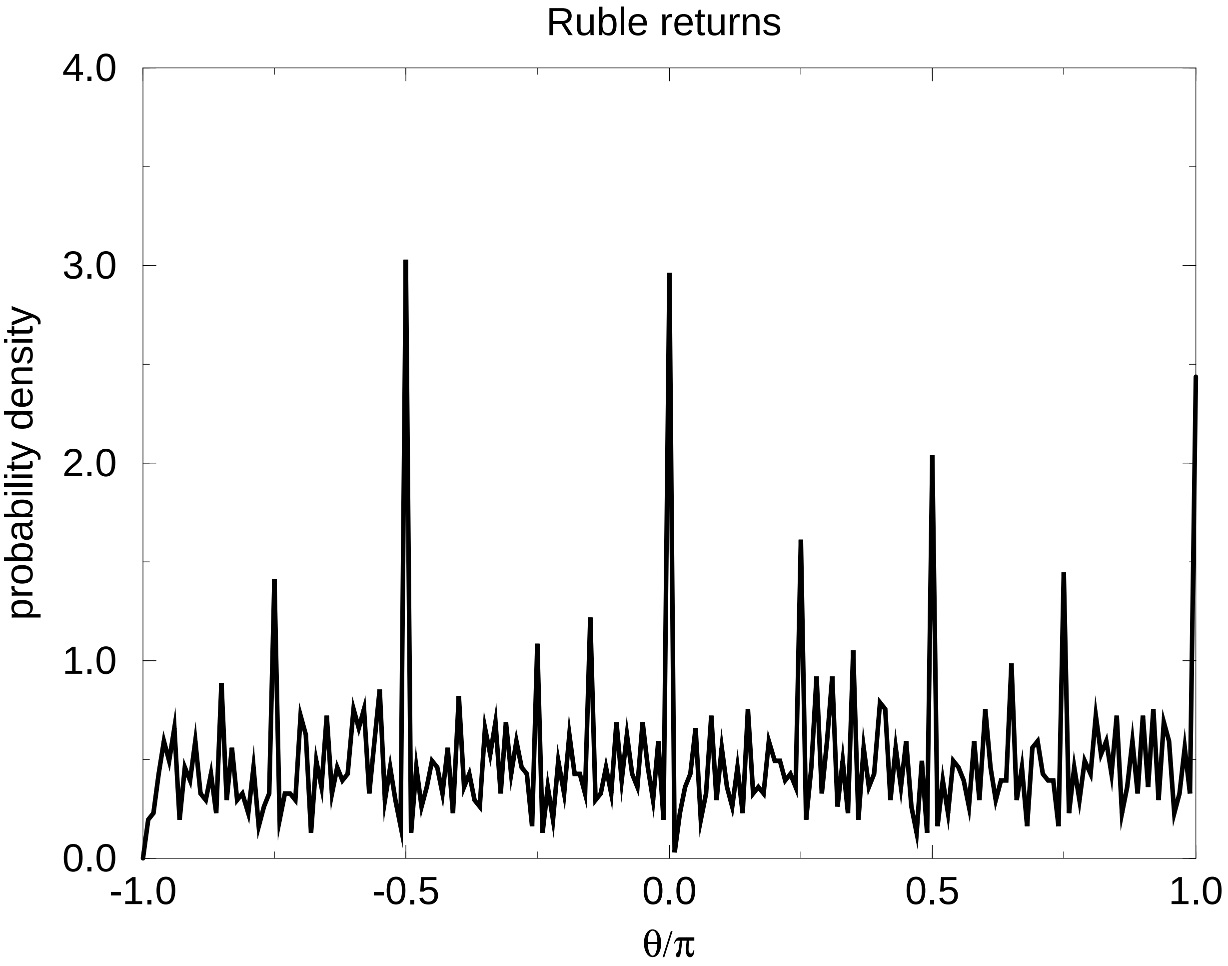

Ruble returns
probability density
4.0
3.0
2.0
1.0
0.0
-1.0
-0.5
0.0
0.5
1.0
θ/π

Fig.2

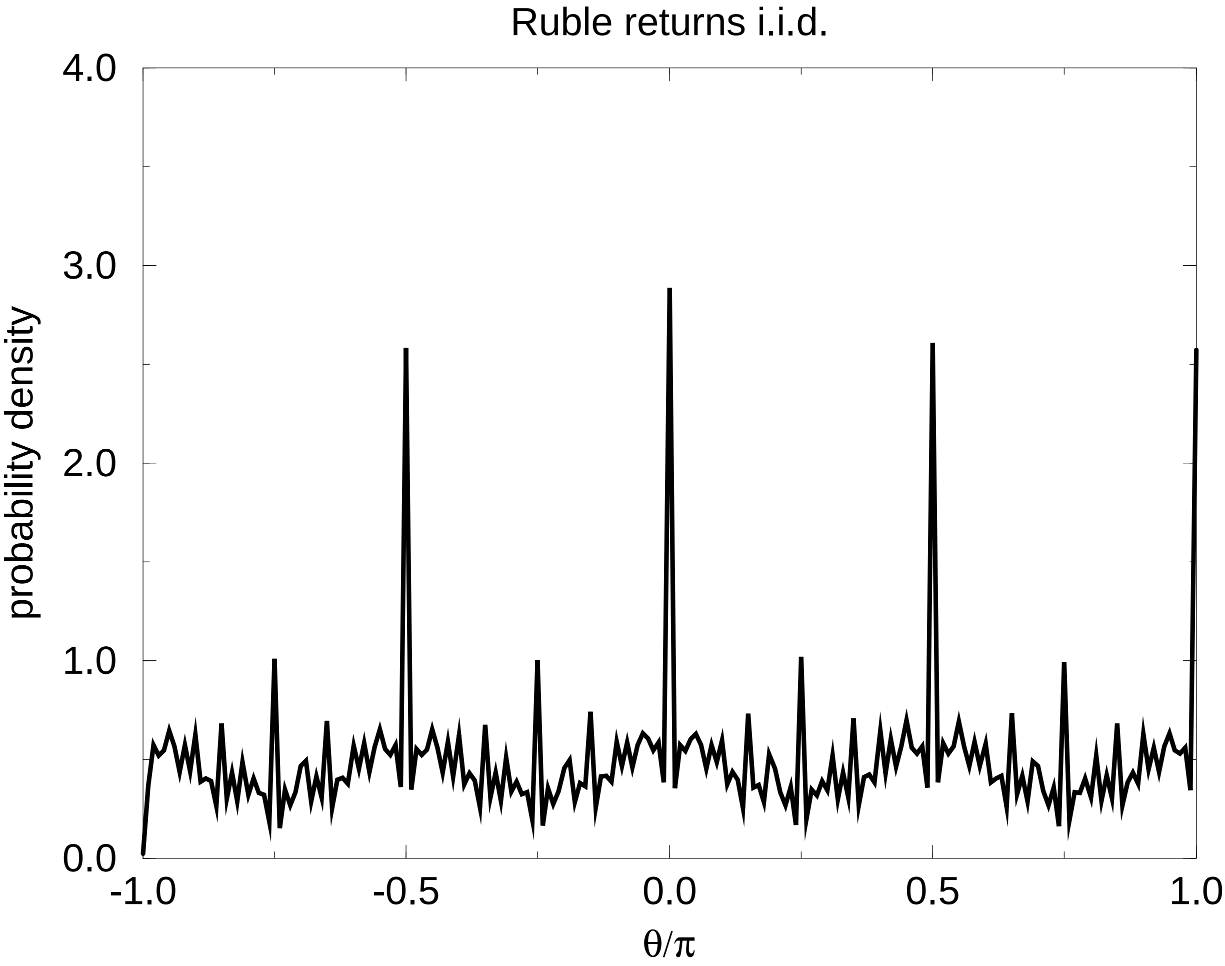

Ruble returns i.i.d.
probability density
4.0
3.0
2.0
1.0
0.0
-1.0
-0.5
0.0
0.5
1.0
θ/π

Fig.3

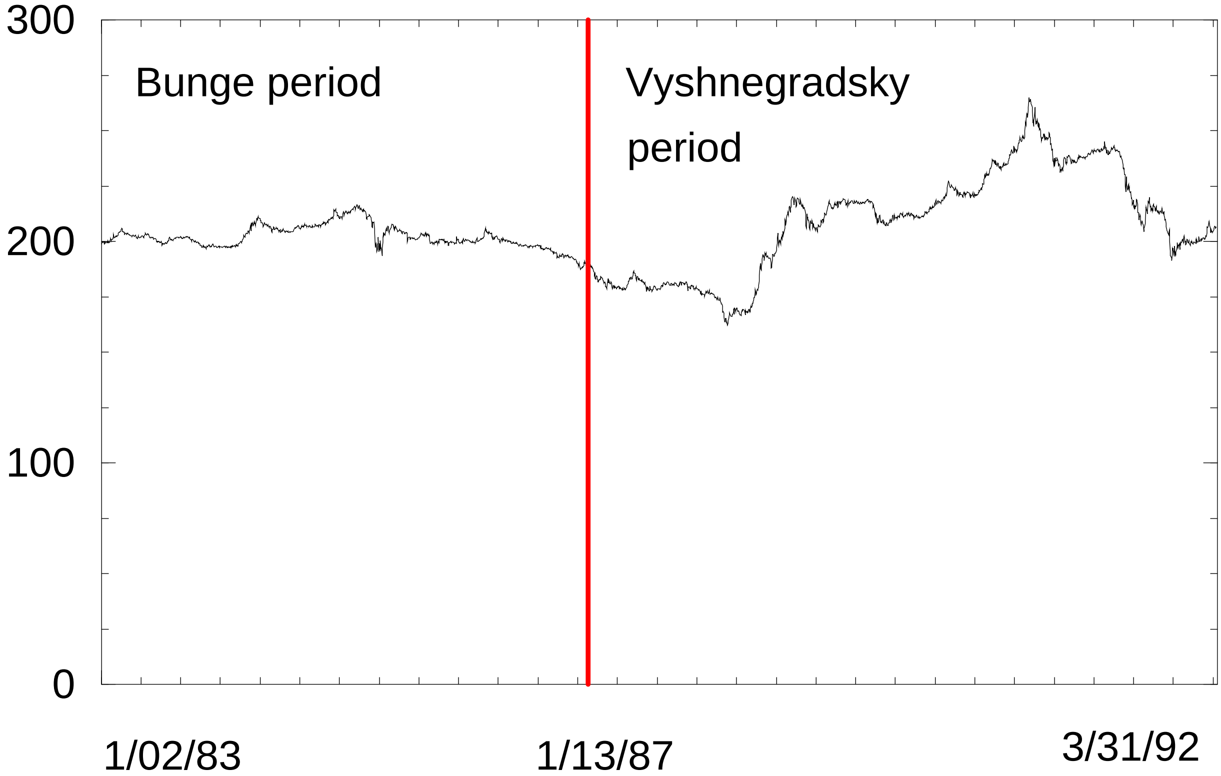

300
200
100
0
Bunge period
Vyshnegradsky
period
1/02/83
1/13/87
3/31/92

Fig.4

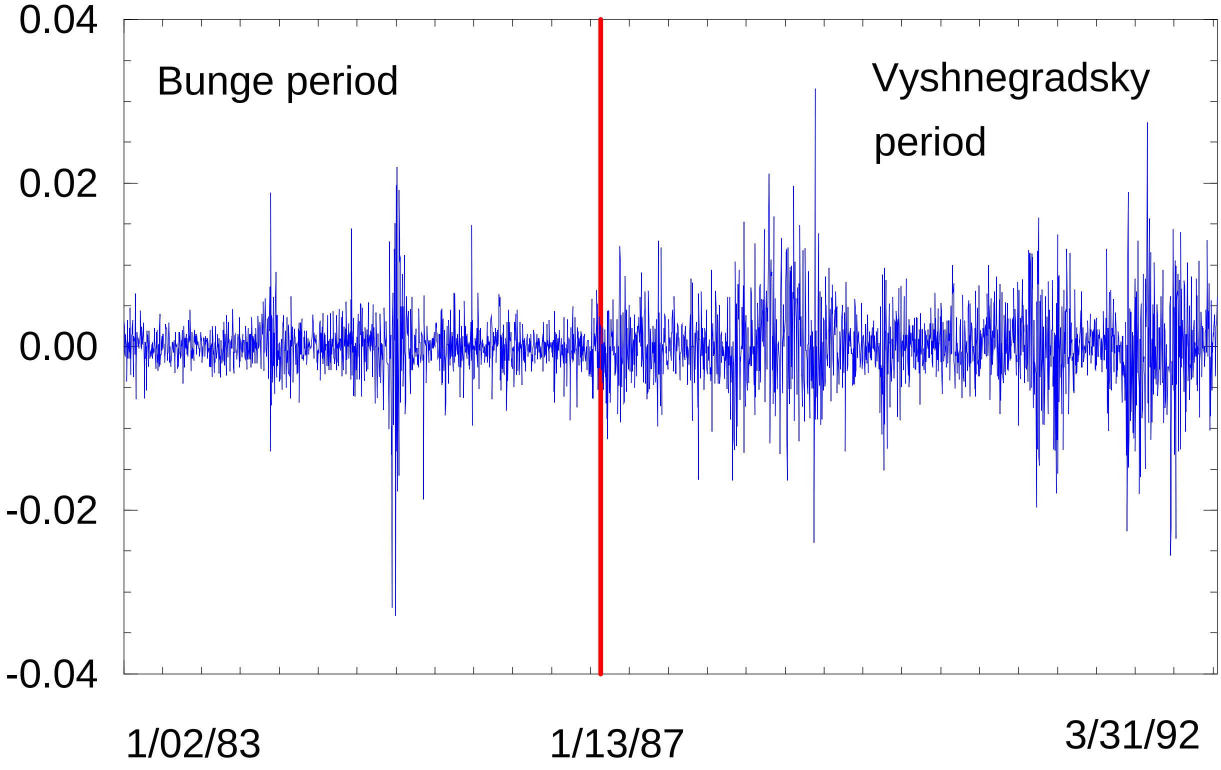
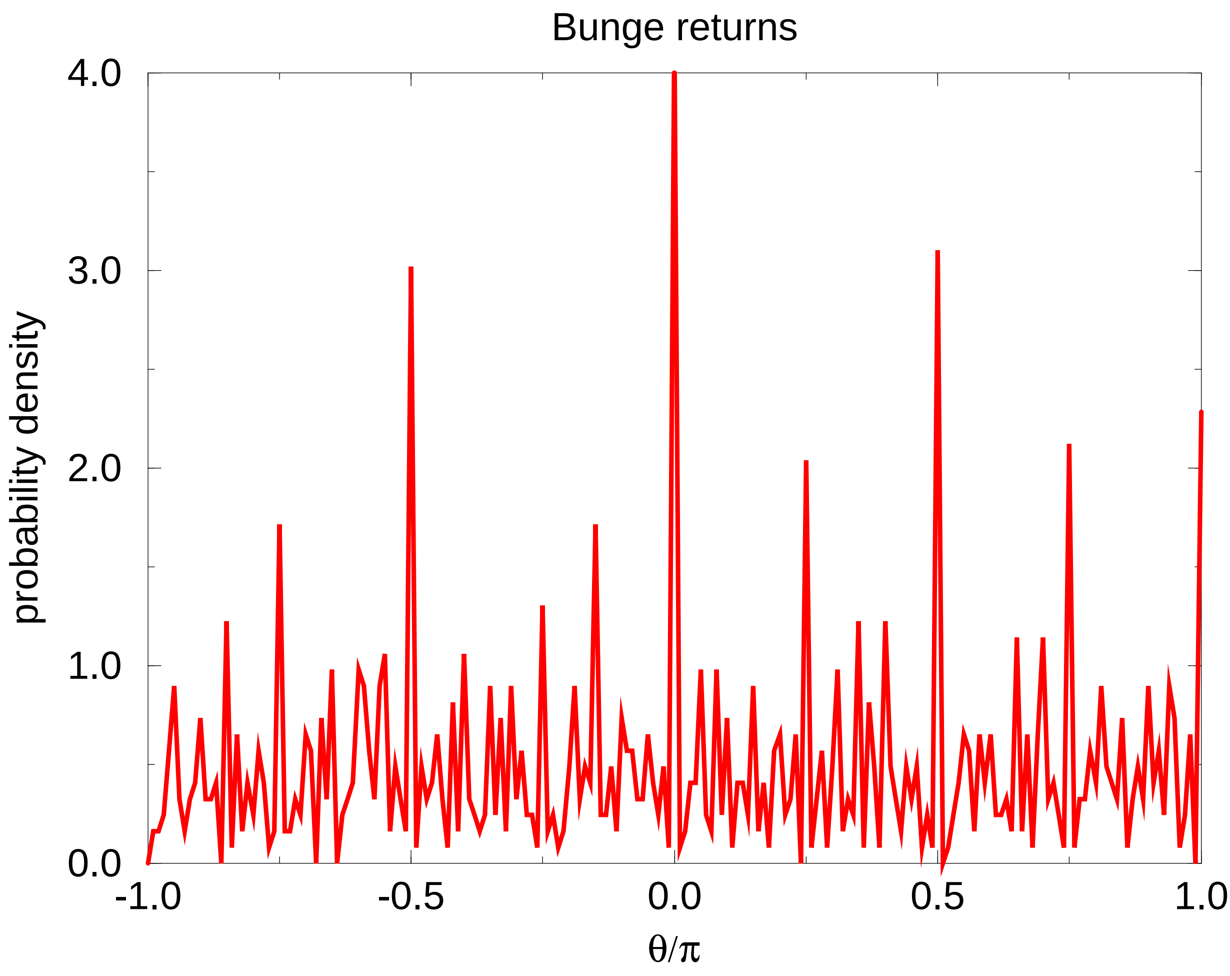

0.04
0.02
0.00
-0.02
-0.04
Bunge period
Vyshnegradsky
period
1/02/83
1/13/87
3/31/92

Fig.5



Bunge returns
probability density
4.0
3.0
2.0
1.0
0.0
-1.0
-0.5
0.0
0.5
1.0
θ/π

Fig.6

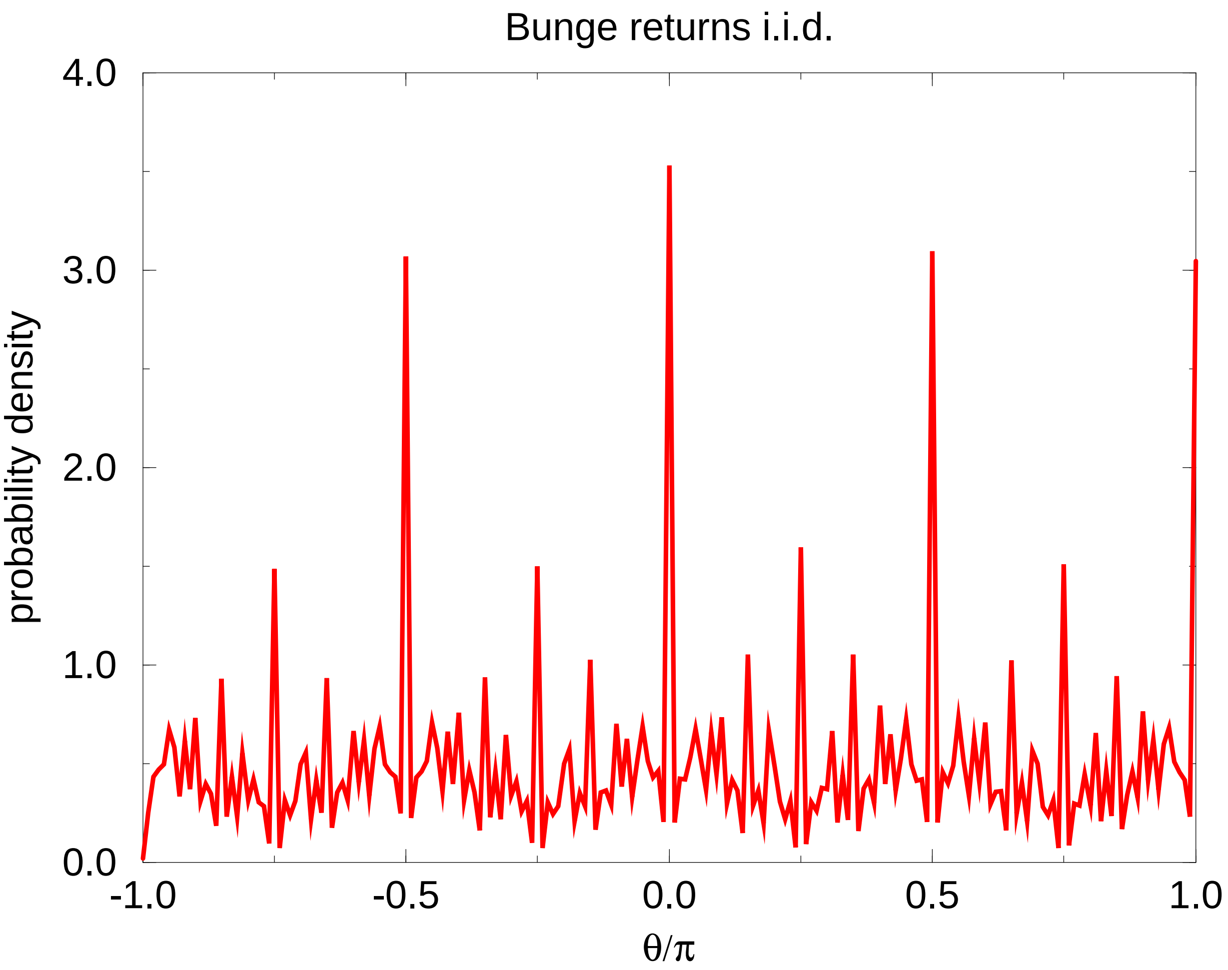

Bunge returns i.i.d.
probability density
4.0
3.0
2.0
1.0
0.0
-1.0
-0.5
0.0
0.5
1.0
θ/π

Fig.7

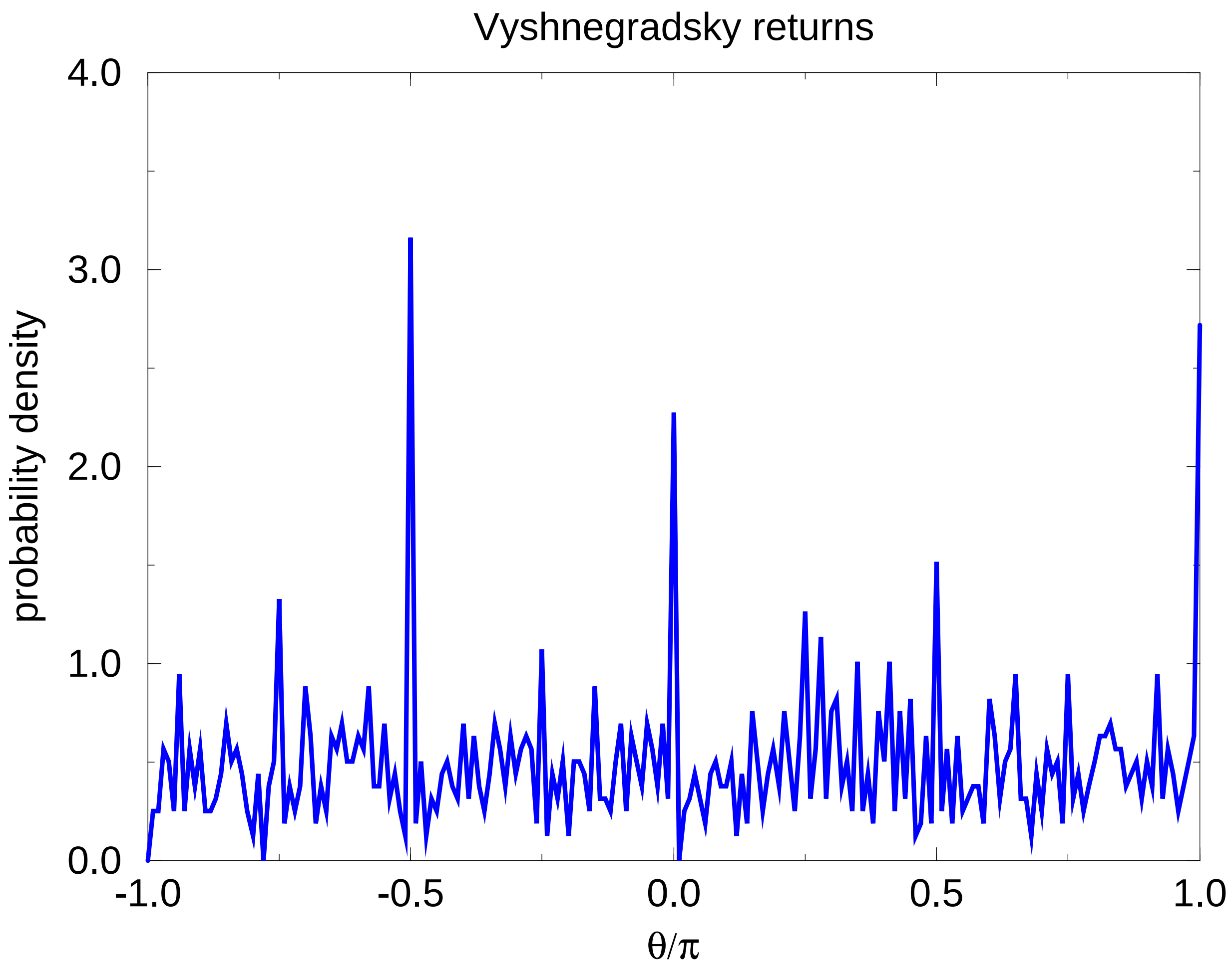

Vyshnegradsky returns
probability density
4.0
3.0
2.0
1.0
0.0
-1.0
-0.5
0.0
0.5
1.0
θ/π

Fig.8

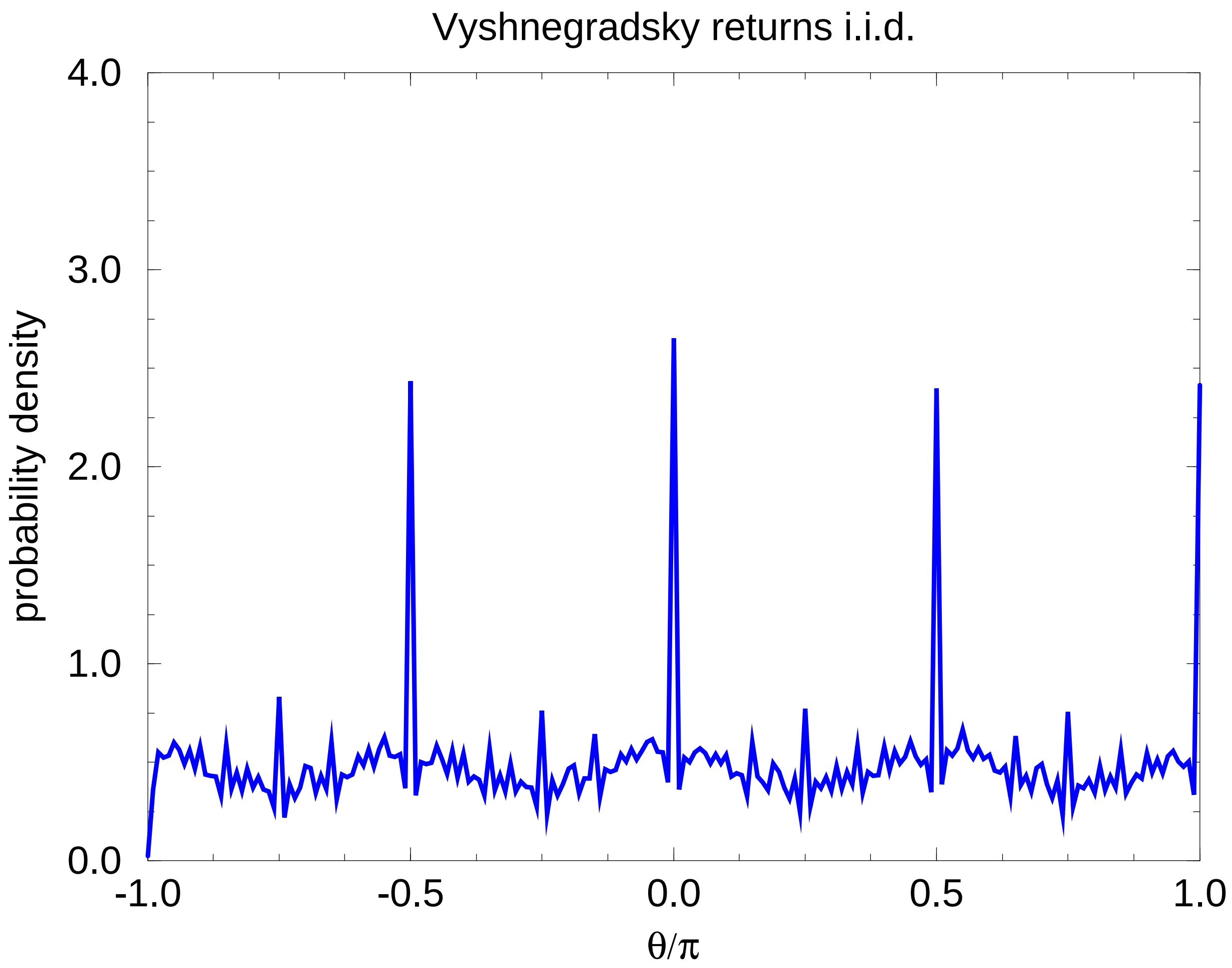
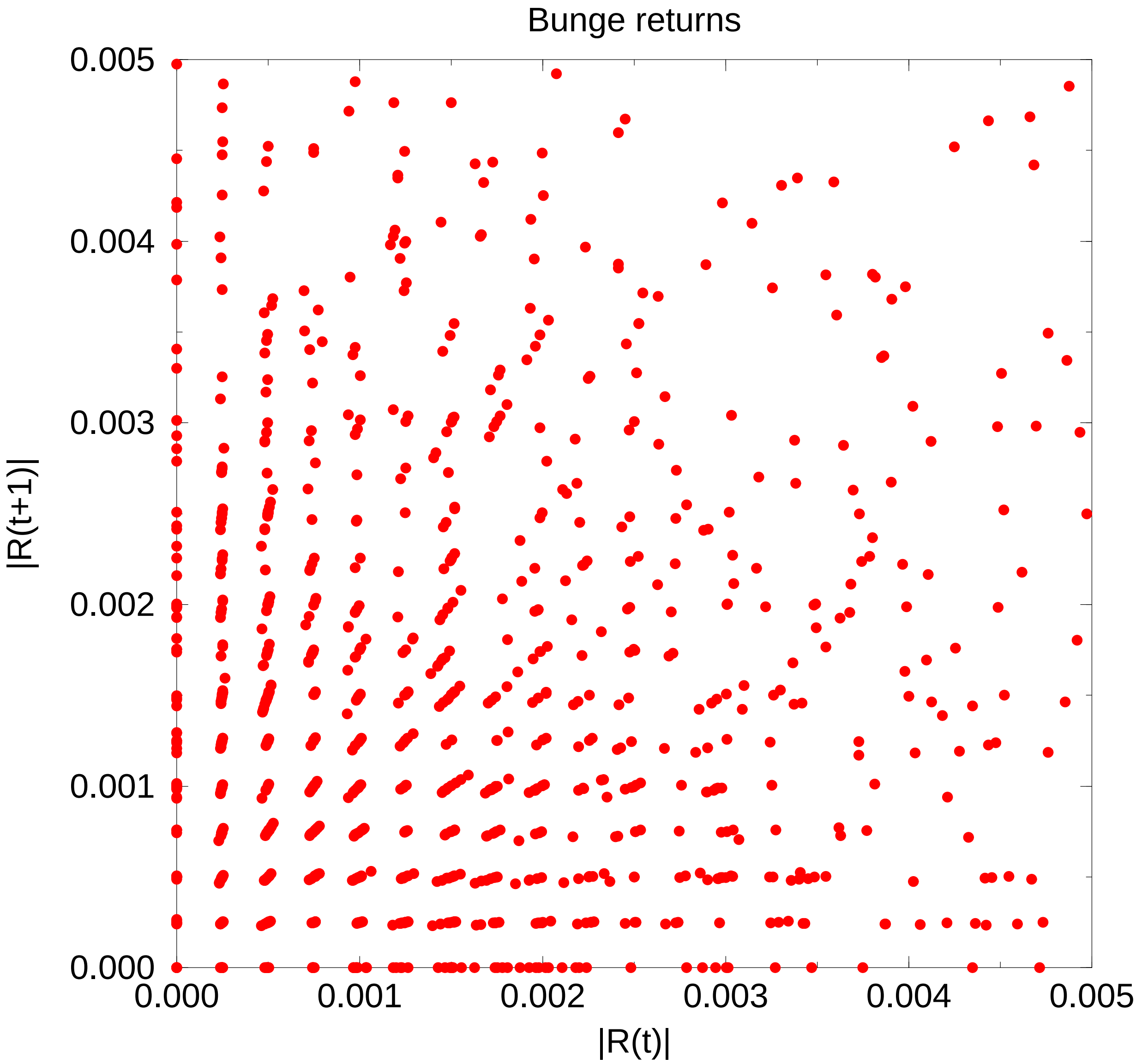

Vyshnegradsky returns i.i.d.
probability density
4.0
3.0
2.0
1.0
0.0
-1.0
-0.5
0.0
0.5
1.0
θ/π

Fig.9



Bunge returns
0.005
0.004
0.003
0.002
0.001
0.000
|R(t+1)|
0.000
0.001
0.002
0.003
0.004
0.005
|R(t)|

Fig.10

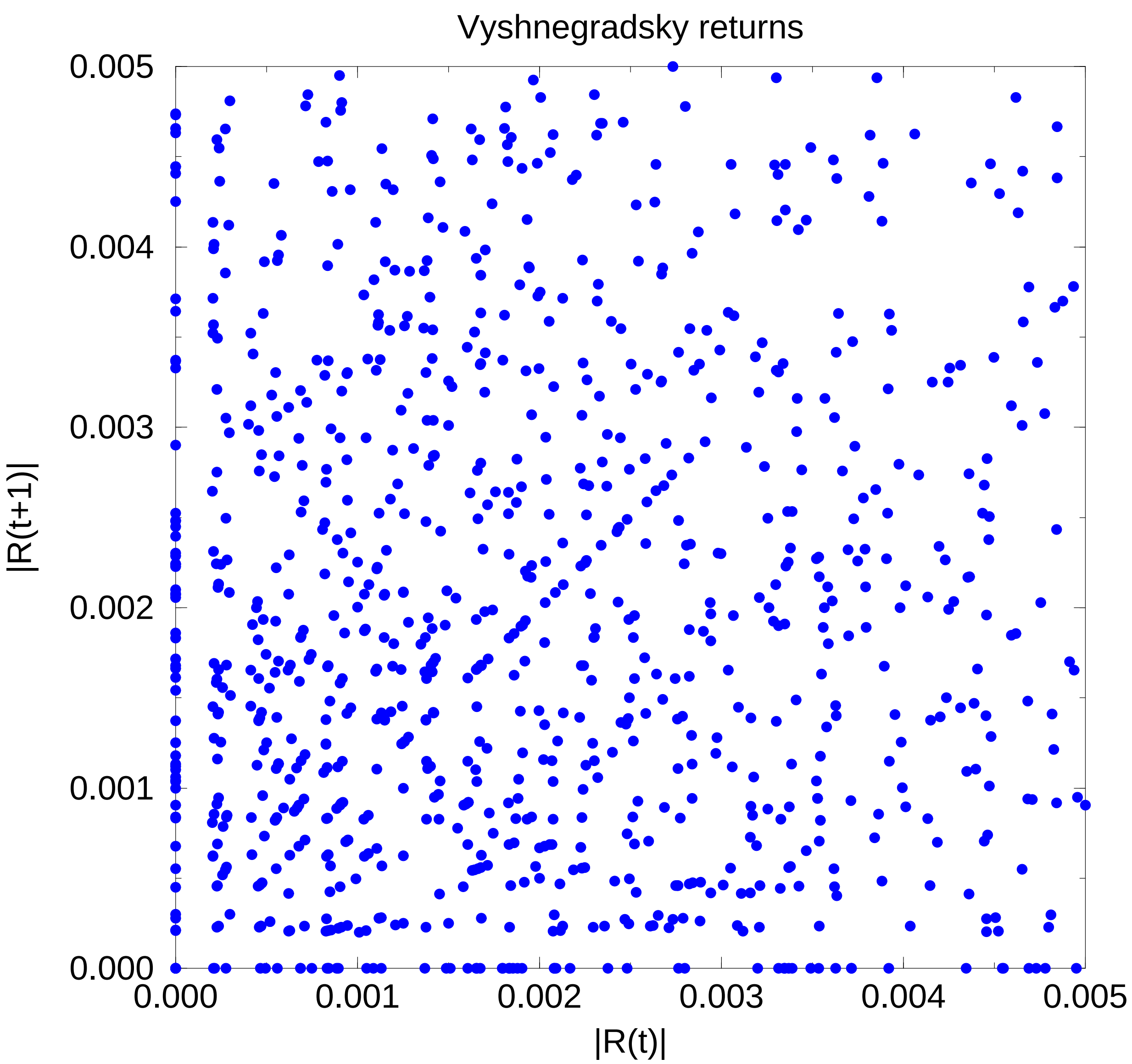

Vyshnegradsky returns
0.005
0.004
0.003
0.002
0.001
0.000
|R(t+1)|
0.000
0.001
0.002
0.003
0.004
0.005
|R(t)|

Fig.11