ABSTRACT
Rotordynamic analysis is an important step in the design of any rotating machine. To go beyond the very simple models yielding a good qualitative insight but cannot predict the details of the dynamic behaviour of rotors, it is necessary to resort to numerical methods and among them the Finite Element Method is without doubt the most suited for implementation in the context of computer aided engineering. Instead of resorting to general purpose codes, the particular characteristics of rotordynamic analysis make it expedient to use specialised tools like DYNROT, a FEM code which allows to perform a complete study of the dynamic behaviour of rotors. Although initially designed to solve the basic linear rotordynamic problems (Campbell diagram for damped or undamped systems, unbalance response, critical speeds, static loading), it has been extended to the study of nonstationary motions of nonlinear rotating systems [1] and the torsional and axial analysis of rotors and reciprocating machines. Its distinctive features of resorting to Guyan reduction and of extensively using complex co-ordinates both for isotropic and non symmetric systems, allow to reduce the computer time and to perform a large number of computations at a reasonable cost. The code can thus be used as a routine to be called by optimisation procedures aimed at including rotordynamic performances into the definition of an optimum design of the machine.

INTRODUCTION
Rotordynamic considerations play an important role in the design of a number of rotating machine elements, particularly when rotational speeds are high. Dynamic analysis, in the past mostly aimed only to the computation of the critical speeds, but currently in many instances used to obtain a whole picture of the rotordynamic behaviour of the machine (e.g., the computation of the Campbell diagram and the unbalance response), must accompany stress analysis and all the other computations related to the working conditions of the machines (e.g. fluid dynamic study for turbomachinery, electrical analysis for generators and motors, etc.).

There are cases in which rotordynamic analysis deeply influences the design of the rotating parts of the machine: an example is the case of composite material transmission shafts for vehicular applications in which the diameter of the shaft and the orientation of the reinforcing fibres are determined by the need of avoiding the presence of a critical speed within the working range [2], or of high speed turbomolecular pumps on magnetic bearing in which the need of rising the third critical speed, the one related to the deformations of the rotor, dictates many design details. In this cases, if an optimisation of the design is attempted, rotordynamic analysis enters deeply the optimisation process.

Traditionally, rotordynamic analysis of complex machines was performed either using much simplified models or through procedures similar to Myklestat-Prohl method, based on the transfer matrices approach. The finite element method (FEM) is presently gaining popularity also in the field of rotordynamics, mainly for its ability of modelling intricate geometries in a simple way, at least from the point of view of the user, and its possibility of writing general purpose codes. However, although the use of standard commercial codes for structural dynamic analysis also for the dynamic study of rotors is sometimes possible, it compels to resort to some sort of “tricks” to
take into account the effects of the rotation of the system, which can have a strong influence on its flexural vibration, affecting the natural frequencies and coupling the flexural motions in such a way that it is more correct to speak of whirling motion than of vibrations.

To take into account this instance, it is possible to force a gyroscopic matrix, which affects mainly the mass matrix of the system, into any code devised for dynamic analysis, but this is not a handy procedure and it has the disadvantage of allowing only the study of synchronous whirling. It is then possible only the computation of the critical speeds and the unbalance response, but not that of the Campbell diagram, i.e. the plot of the whirl natural frequencies against the spin speed, which is the basic tool for understanding the dynamic behaviour of any rotor. Even more problematic is the study of the behaviour of the accelerating rotor. Only a purposely written code, which takes correctly into account the presence of the gyroscopic matrix and of the circulatory matrix linked with rotating damping and perhaps of the centrifugal stiffening effect, due to the presence of bladed disks, can fulfil adequately the task.

Starting from the end of the seventies, the development of a FEM code specifically suited for rotordynamic computations was undertaken at the Department of Mechanics of Politecnico di Torino. It has been evolved in various versions through more than twenty years [3]. The original code was written using HPL and then HP-BASIC language for desktop HP 9800 computers but the present versions are based on the MATLAB (MATLAB is a trademark of The MathWorks, Inc.) interactive software package and can be used on any PC, workstation or mainframe system.

THEORETICAL BACKGROUND

Usually rotors are modelled as beam-like structures and some form of beam theory, with either the Euler-Bernoulli or the Timoshenko approach [4, 5] is used. Numerical solutions based on some form of the Myklestadt-Prohl transfer matrix method [6, 7] were very widespread. Also most of the procedures based on the FEM consider the structural parts as composed by beam elements [8, 9]; although models based on other types of elements can be found the literature [10]. Also for the study of the torsional behaviour of shafts the structural parts are mainly modelled as beams or even simple torsional springs.

Under the above mentioned assumptions, if the beams which model the rotor are straight, their axes are all aligned with the spin axis and if the centre of gravity and the shear centre of all the cross sections lay on the spin axis, the axial, flexural and torsional behaviours are uncoupled. The presence of a small static or couple unbalance does not modify substantially this feature. Clearly the assumptions leading to uncoupling do not hold in the case of crankshafts, but in the study of the torsional vibrations of reciprocating machines it is customary to resort to a so-called equivalent model which is essentially a straight beam-like structure and, as a consequence, the uncoupling is restored [11].

The further assumptions of linearity, small unbalance and small displacements allow to obtain a linear equation of motion; however, even in the case of the discretised model of a linear rotor which is axially symmetrical about its spin axis and rotates at a constant spin speed \( \omega \), the linearised equation of motion is of the type

\[
M \ddot{x} + (C + G) \dot{x} + (K + H) x = f(t),
\]

where \( x \) is a vector containing the generalised co-ordinates, referred to an inertial frame, \( M \) is the symmetric mass matrix, \( C \) is the symmetric damping matrix, \( G \) is the skew-symmetric gyroscopic matrix (it is usually linearly dependent on the spin speed \( \omega \)), \( K \) is the symmetric stiffness matrix (it may contain a part which is proportional to \( \omega^2 \)), \( H \) is the skew-symmetric circulatory matrix, usually proportional to \( \omega \) and \( f \) is a time-dependant vector in which all forcing functions are listed. One of these forcing functions is usually that due to the residual unbalance which, although small,
cannot be neglected. Unbalance forces are harmonic functions of time, with amplitude proportional to $\omega^2$ and frequency equal to $\omega$. Equation (1) is that of a non-natural, circulatory system and hence differs from the typical equations encountered in dynamics of structures, where all matrices are symmetric. It must be noted that, when $\omega$ tends to zero, the skew-symmetric terms vanish and the rotor reduces to a structure.

Equation has been obtained assuming that the rotor is axially symmetrical, but it runs on a stator which can be without any particular symmetry properties. If, on the contrary, the rotor cannot be considered axially symmetrical, the study becomes very complicated, unless an axial symmetry assumption can be made on the nonrotating parts of the system. In the latter case, a rotor-fixed reference frame, i.e. one that rotates at the angular velocity of the rotor, can be used and an equation similar to equation (1), although written with reference to a non-inertial frame is obtained. If both stator and rotor are non-isotropic with respect to the rotation axis, the equation of motion which models its behaviour has coefficients which are periodic in time, with a frequency equal to $2\omega$. No closed form solution of such equation is available and even to reach approximated solution is far more complicated than in the case in which either the rotor or the stator is axially symmetrical.

When the flexural behaviour can be uncoupled from the axial and torsional ones, equation (1) holds for the first one, while the torsional and axial equations of motion are usually those of a natural, non-circulatory system.

If both stator and rotor are isotropic with respect to the rotation axis, the best choice for what the generalised co-ordinates for the study of the flexural behaviour are concerned is the use of complex co-ordinates. As the structure is assumed to be beam-like, each node has four real degrees of freedom, namely two lateral displacements and two rotations. Assuming that the axis of rotation of the system coincides with the $z$-axis of an orthogonal reference system $xyz$ and using the assumptions which are customary in the beam theory, the flexural displacement of any point of the rotor axis and the rotation of the cross sections can be expressed by the complex displacement

$$
\begin{cases}
z = x + iy \\
\varphi = \varphi_y - i\varphi_x .
\end{cases}
$$

When using complex co-ordinates for a system which is axially symmetrical, Equation (1) yields:

$$M \ddot{\mathbf{q}} + (C + iG) \dot{\mathbf{q}} + (K + iH) \mathbf{q} = \mathbf{f}(t),
$$

where $\mathbf{q}$ is the vector containing the complex generalised co-ordinates. Note that when using complex co-ordinates all matrices are symmetrical: those which in the standard approach are skew-symmetric, in the present case are imaginary.

It is possible to demonstrate that the complex-coordinate approach is suitable also when the rotor or the stator lack of axial symmetry [12]. By releasing also the linearity and constant speed assumptions, the general equation of motion of a nonlinear rotor which is performing an acceleration with a stated law $\omega(t)$ is [13]:

$$
\begin{align*}
M_m \dddot{q} + &\left( C_m - i\omega G \right) \ddot{q} + \left( K_m + K_{\omega} \omega^2 - i\omega C_m \right) \dot{q} + M_{nd} \dddot{q} + M_{rd} e^{2i\omega} \ddot{q} + C_{nd} \dddot{q} + \left( C_{rd} + 2i\omega M_{nd} \right) e^{2i\omega} \ddot{q} + \\
K_{nd} \dddot{q} + &\left( K_{rd} - i\omega C_{rd} \right) e^{2i\omega} \ddot{q} + g(q, \dot{q}, \ddot{q}, \dot{\omega}(t)) = (\omega^2 - i\dot{\omega}) F_x e^{i\omega} + F_y,
\end{align*}
$$

where:

- $M$, $G$, $C$, $K$ and $K_\omega$ are respectively the mass, gyroscopic, viscous damping stiffness and centrifugal stiffening matrices. They are all symmetrical and have been written in such a way to evidence the dependence of the various matrices (or parts of matrices) from the spin speed $\omega$;
- subscripts $m$, $d$, $r$ and $n$ refer respectively to the mean and deviatoric matrices (for their definition, see [11]) and to the matrices referred to the rotating and the non-rotating parts of the machine. Mean matrices without either subscript $r$ or $n$ refer to the whole model;
• $q$ and $F$ are respectively the vector of the complex generalised co-ordinates and that of the nodal forces. In this case subscript $r$ designates forces due to unbalance and subscript $n$ the non rotating forces as rotor weight. All forces can be prescribed functions of time;

• the generic vector function $g(q, \dot{q}, \dot{\theta}(t))$ is introduced to take into account the behaviour of the nonlinear part of the system;

• $\dot{\theta}$, $\omega = \hat{\theta}$ and $a = \hat{\omega}$ are respectively the rotation about the spin axis, the spin speed and the angular acceleration. They are all known functions of time and are the same for the whole rotor; however with simple modifications to equation (4) to study the behaviour of multi-shaft systems, the model can be subdivided into different substructures each one having a different angular velocity and hence different laws $\dot{\theta}(t)$, $\omega(t)$ and $a(t)$.

The assumption that the angular acceleration is the same at all sections comes directly from the uncoupling between the torsional and the flexural behaviour and is usually referred to as torsionally stiff rotor assumption. However, an equation allowing to link the rotational degree of freedom of the system with the driving torque can be easily obtained. It is a single scalar equation, owing to the assumption of torsionally rigid rotor, except in the case of multi-shaft systems

\[ M_z = \sum [F^T \hat{q} \hat{q}^T] + J_p, \]  

(5)

where the total polar moment of inertia of the rotor can take also into account the effect of the different unbalances which are at any rate very small.

Equation (4) is the basic formulation for the study of the flexural behaviour of the system. Clearly its complete formulation cannot be solved in closed form and the only possible approach is the numerical integration in the time domain.

When the forcing functions are periodic, the system is linear, the spin speed $\omega$ is constant and either the stator or the rotor (or both) can be considered as axially-symmetrical bodies with respect to the spin axis, general solutions in the frequency domain can be obtained in closed form.

By assuming that a solution of the type of $q = q_0 e^{\omega t}$ or $q = q_0 e^{\omega t + \pi}$ respectively for free whirling and synchronous forced whirling (i.e. a forced whirling due to a forcing function whose frequency is equal to the spin speed, as in the case of unbalance), simple frequency domain equations allowing to solve the typical problems of rotordynamics are readily obtained. The critical speeds can thus be computed, of the Campbell diagram, the stability maps and the roots loci can thus be obtained through an eigenanalysis, and the steady state response to unbalance or other periodic forcing functions can be obtained.

While in frequency-domain equations damping can be modelled using both the viscous and hysteretic models, when solving equation (4) in the time domain only viscous damping can be introduced. Viscous damping can be introduced in form of damper elements and hysteretic damping by stating the loss factor of the materials. When hysteretic damping can be used directly, the equation of motion in the frequency domain can be accordingly modified [11].

When on the contrary this is impossible the user has two alternatives: neglect hysteretic damping altogether or resorting to a form of equivalent viscous damping. A way to compute the latter is that of performing the modal transformation based on the modes of the linearised, natural, isotropic system (i.e. based only on the mass and stiffness mean matrices) of all hysteretic damping matrices and reducing them to their generalised proportional component [11] by cancelling all elements outside the main diagonal. The equivalent viscous damping matrices can then be obtained by dividing the elements on the main diagonal by the corresponding natural frequency and then performing the inverse modal transformation. This procedure is clearly approximated, but, if the
system is lightly damped, leads to acceptable results. A better procedure, which takes into account also the non-natural nature of the system is being developed and will be published soon. There is no theoretical way to assess the precision of such approximated procedures, but a check can be done by comparing the steady-state unbalance responses of the models with hysteretic and with viscous equivalent damping. Note that the equivalent damping so computed is just a mathematical approximation and the structure of the equivalent damping matrices has nothing to do with that of the starting hysteretic matrices, nor it retains any physical meaning.

In the study of the unbalance response, both at constant speed or during an acceleration, equation (5) may be used to compute the driving torque needed to maintain the spin speed constant or to follow the stated law $\omega(t)$, after the lateral behaviour of the system has been obtained.

The main advantages of the use of complex co-ordinates can be summarised as follows:

- in the case of isotropic systems, the equations are all real and their number is halved with respect to the approach based on real co-ordinates. A substantial reduction of computation time is so achieved;
- when the system is not isotropic, elliptical or polyharmonic whirling is expressed as the sum of forward and backward circular whirling components. When more complicated whirl patterns exist, the various harmonic components are well separated and this allows to obtain a good insight of the motion of the system in a simpler way;
- the model is built in terms of mean and deviatoric properties. Different solutions can then be obtained one after the other: by neglecting the deviatoric matrices the dynamic behaviour of a mean system, i.e. an equivalent symmetric system which retains many of the features of the original one, is obtained. Later more realistic simulations which take into account deviatoric matrices can be obtained without the need of building new models. The possibility of refining the solution without the need of re-modeling the system is a very interesting feature of this approach.

The study of the dynamic behaviour of reciprocating machines is performed using the equivalent system approach described in detail in [11]. The so-called inertia torques are directly introduced into the model, together with the forcing functions defined by the user. Also here hysteretic damping can be directly introduced into the model only in some of cases. When this is impossible, an equivalent viscous damping matrix can be computed following the lines seen for flexural behaviour.

**DYNROT CODE**

The code is implemented in the form of function and script M-files, all written in MATLAB language However, the user needs only a basic knowledge of the language to run DYNROT.

The user may define the model either through an interactive input routine or by preparing a data m-file, but the most interesting feature, particularly when performing an optimization, is the possibility of using the code in parametric form. All characteristics of the elements can be introduced in form of parameters, which can be defined in the calling instruction of an external code, of which DYNROT can be thought as a subroutine. There is no limit to the number of parameters, which can be used, allowing to prepare a general model for a class of rotating machines more than for a particular rotating system.

For simple non-parametric computations the user can chose to run the code in an interactive way, but in general it is possible to prepare a driver file to run the code in a batch way or, as already stated, to call DYNROT from a code which defines the parameters and performs the optimization.
As an example, if a genetic algorithm is used to optimize the shape of a rotor and some rotordynamic feature is included into the fitness function, the code in which the genetic algorithm is implemented can simply call DYNROT passing to it all the parameters required to define the rotor (which are defined by the genoma of the particular individual machine whose fitness function must be computed) and getting back the relevant rotordynamic features. The ability of DYNROT of performing the required analysis in a very short time is an essential feature in the context of genetic algorithms, as the number of individual configurations studied can be very high.

Apart from the results related to the dynamic analysis, while building the model, the inertial properties (mass, moments of inertia and co-ordinates of the centre of mass) of the whole model and of the various substructures are computed and a drawing of the model can be supplied.

In the present version of the code time-domain numerical integration is performed using 4-th order Runge-Kutta adaptative algorithm and nonlinear algebraic equations are solved using Newton-Raphson algorithm. Equations with time-depending coefficients are solved by resorting to an approach similar to Hill's infinite determinant, obviously truncated at a certain harmonic at the choice of the user, while linear sets of equations and eigenproblems are solved using the facilities which are standard in MATLAB package. In all cases, when complex arithmetic is needed, the ability of MATLAB to deal with complex quantities is exploited, the only exception being equation in which both a complex unknown and its conjugate are present which need to be treated by separation of the real from the imaginary part.

The element library of DYNROT code is particularly tailored for rotordynamics applications; in the present version 23 mechanical elements and 2 control elements are included. All beam elements are based on a formulation of the type usually referred to as "simple Timoshenko beam" with consistent formulation for mass and gyroscopic matrices. Spring and damper elements can connect two nodes of the structure (e.g. to simulate joints, bearings or dampers between two different shafts or between a shaft and the stator) or one node to a fixed point (e.g. to simulate an elastic support). If two nodes are present their co-ordinates must be the same. Also a cubic spring element and a linear spring with clearance, useful to model respectively ball bearings or elastomeric springs and roller bearings, are present.

Bladed discs can be modelled using six different elements for discs and rows of blades, including different types of transition elements to connect discs with shafts, modelled as beams. They are basically annular elements in which the displacement field is approximated by algebraic polynomials in radial direction and trigonometrical polynomials along the angle. As only the first two terms of the polynomial affect the dynamics of the rotor as a whole (the following ones affect the local dynamics of the bladed disc), the polynomials are truncated after two terms [14], [15].

Hydrodynamic bearings are modeled using the conventional 8-coefficient model, with either constant or speed-dependent parameters, following the short, fully cavitated, bearing assumption [11] or by entering the coefficients as functions of the Sommerfeld number. All tables reported in [16] are included, other types of bearing may be entered by the user.

The magnetic bearing elements can be connected at both ends at two nodes of the structure for the electromagnetic actuator and at two nodes for the sensor or at one node only, the other end being fixed, i.e. connected with the ground. The characteristics of the bearing can be isotropic in $xy$ plane or anisotropic, which allows to study the cases of bearings which are geometrically and electrically isotropic but work in different conditions in $xz$ and $yz$ planes due to the presence of static forces. The element includes the actuators, two electromagnets in $x$ and $y$ directions, which supply a force
proportional to the square of the current \( i_c \) and inversely proportional to the square of the radial displacement \( u \):

\[
F = -k \left( \frac{i_c}{u} \right)^2
\]

and two displacement sensors measuring the displacements in the same directions. The sensors may be located in nodes different from those in which the actuators are located (non-colocated bearings). A bias current can be given to linearize the characteristics of the actuators.

As active magnetic bearings need a suitable control system, two controller elements are included. The first is a PID controller on error feedback with an additional filter on sensor output. Its frequency-domain transfer function takes into account the pole necessary to make the derivative filter feasible

\[
\text{PID}(s) = k_c \left( 1 + \frac{1}{sT_i} + \frac{T_ds}{N} \right),
\]

where \( k_c \) is the overall stationary gain, \( T_i \) the reset time, \( T_d \) the prediction or derivative time and \( N \) the ratio between zero and pole of the PD section. The filter on sensor output has the following transfer function

\[
F(s) = \frac{1}{s\tau + 1},
\]

where \( \tau \) is its time constant.

Also a general controller is included. Its characteristics are expressed in the form of the coefficients of the polynomials at numerator and denominator of the transfer function. The general controller transfer function \( C(s) \) has the following structure

\[
C(s) = \frac{b_n s^n + b_{n-1} s^{n-1} + b_{n-2} s^{n-2} + \ldots + b_0}{s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + \ldots + a_0}.
\]

An element which can be used to model the inertial properties of a crank mechanism for the study of crankshafts is included [17]; it has only one node, as a mass element. The elastic properties of the crank can be introduced in the form of a spring element, having the correct equivalent stiffness, or of a beam element with a suitable equivalent length.

DYNROT code works for most of the computations using directly physical generalized coordinates. In particular, the torsional response to a polyharmonic forcing function do not rely on any modal approximation, as the usual procedure which takes into account only the response related to the first or the few first modes can lead to unacceptable approximations. However, there are cases in which the modal approach allows strong reduction in computation time and consequently is worth while considering. In the flexural behaviour two different modal approaches are possible; the first is that of using as transformation matrix the matrix of the eigenvectors of the natural, undamped, linearized, isotropic system while the second uses the right and left eigenvectors of the non-natural system [18]. The modal approach (first of the above mentioned procedures) is compulsorily used for the computation of the acceleration response. To avoid high frequency modes which would cause problems in the numerical integration procedure, only the modes with natural frequency not higher than the maximum spin speed are considered.

Another case in which the modal approach is used is the study of rotors including magnetic bearings. Here the user can decide the number of modes to be considered, being possible to chose all modes if the approximation due to the modal approach is considered unacceptable.

Guyan reduction is extensively used to reduce the size of the problem. It allows to obtain results...
which are very close to the correct ones even with a small number of master degrees of freedom if their choice is performed correctly. The choice of the degrees of freedom to be considered as slave is mainly a fact of experience and physical insight of the problem; different reduction schemes can be evaluated to obtain best results.

EXAMPLE 1
As a first example, consider a typical problem in rotordynamics: the influence of the bearing stiffness on the critical speeds of a rotor. In the case of a rigid rotor, the problem is easily solved by using a simple model with 4 degrees of freedom, but this approach does not allow to study the effect of modes linked with the deflection of the shaft, i.e. while being adequate for compliant bearings, does not allow to study the effect of very stiff supports. Using DYNROT it is possible to model also the compliance of the rotor and to perform a complete study.

Consider as an example the small gas turbine studied in [11] as Example 4-3 and modeled using 11 nodes and 14 elements (including 2 mass elements and 2 spring elements to model the bearings). The model has a total of 22 degrees of freedom reduced to 8 through Guyan reduction. A plot of the first 4 critical speeds as functions of the stiffness of the bearings (from a minimum of $10^5$ to a maximum of $10^{10}$ N/m) is reported in Fig. 1a. The plot is a pretty much standard result; here the interesting point is the fact that it has been produced in a completely automatic way through a simple MATLAB program of about 25 lines (including all graphic commands) which uses DYNROT code as a subroutine. From the plot is clear that the first two critical speeds and, to a lesser extent, the fourth one, are strongly influenced by the stiffness of the bearings, while the third one is not.

Using a slightly more complicated program it is possible to investigate separately the effect of the stiffness of the two bearings. The result shown in Fig. 1b in terms of a tri-dimensional plot of the first two critical speeds as functions of the stiffnesses of the two bearings $k_1$ and $k_2$, has been obtained by analysing 256 automatically generated FEM models and then by summarising the results in a single plot. The total computation time was 94 s on a Pentium 2 desktop computer.

EXAMPLE 2
As a second example consider a canned pump on magnetic bearings. The machine is intended to be used in the subcritical range with respect to the critical speed related to the first deformation mode,
while being supercritical with respect to the rigid body modes. To reduce the mass of the rotor the central shaft is bored, but the diameter of the bore is critical in that initially its increase raises slightly the critical speed, owing to a reduction of the mass, while then it reduces it owing to a reduction of the stiffness. A plot of the mass of the rotor and of the critical speed as functions of the inner diameter can be obtained using a detailed model of the machine (Fig. 2).

![Critical speed related to the deformation of the rotor and mass of the rotating system of a canned pump as functions of the inner diameter of the rotor.](image)

The plot can be useful to reach a compromise between the contrasting requirements of reducing the mass of the rotor and increasing the critical speed.

**CONCLUSIONS**

Rotordynamic analysis is an essential step in the design of machines containing rotating elements, particularly when high rotational velocities are involved. Current trends in technology towards faster and lighter machines, made using materials with higher performances working at higher stress levels, lead to an ever-increasing need of a detailed knowledge of the dynamic behaviour of the machine as a system. The simple computation of the first critical speed of the rotor is often not enough and design must include a more detailed analysis. Some of the classical assumptions of rotordynamics, like axial symmetry or linearity, have to be dropped, and a more complete models are needed.

Finite element codes are flexible and powerful enough to answer the mentioned needs, but standard FEM codes do not include some of the features which are typical of rotordynamics, like gyroscopic effect, rotating damping and centrifugal stiffening. There is a need for purposely-written codes for rotordynamic analysis, which combine the possibility of performing an analysis of the required completeness with low computer time. The last feature is important particularly when the code is used in the design or optimisation process, when a large number of configurations must be considered.

DYNROT code allows to study the lateral, axial and torsional dynamic behaviour of rotating systems. When the assumptions of linearity, small displacements and rotations and constant spin speed and unbalance are possible it works preferentially in the frequency domain, allowing to reach
general results, while in other cases it performs the time-domain numerical integration of the equations of motion.

The code is essentially a parametric code and can be used as a routine, which is called by optimisation procedures, being immaterial the type of the latter. Extensive use of Guyan reduction allows to obtain models which are simple enough to perform the dynamic analysis a large number of times in a quick and economical way. When dealing with controlled rotors, as in the case of rotors running on active magnetic bearings, it allows to deal also with the parameters of the control loop. Reduced-order models to be used in the design of the control system can thus be readily obtained.

REFERENCES