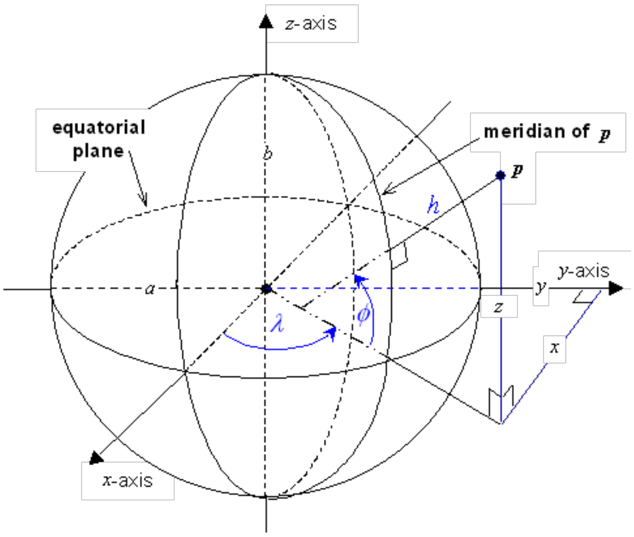
2021_0421 updated

X Y Z ↔ φ λ h

di Michele T. Mazzucato

L’origine del sistema di coordinate cartesiane ortogonali nello spazio X, Y, Z coincide con il centro geometrico dell’ellissoide utilizzato come superficie di riferimento per le coordinate di geografiche ellissoidiche φ, λ, h. L’asse Z coincide con l’asse minore o di rotazione dell’ellissoide ed è positivo verso il Polo Nord, l’asse X giace nel piano equatoriale XY (origine delle latitudini) nell’intersezione con il piano meridiano XZ di Greenwich (origine delle longitudini) ed è positivo verso il meridiano di Greenwich e l’asse Y giace nel piano equatoriale XY, ortogonale all’asse X ed è destrorso positivo.



da https://standards.sedris.org/18026/index.htm

Il *passaggio diretto* dalle coordinate geografiche ellissoidiche φ, λ, h a quelle cartesiane ortogonali nello spazio X, Y, Z non presenta particolari difficoltà. Esso avviene mediante le seguenti relazioni:

$$X = \left(\frac{a}{\sqrt{(1 - e^2 \sin^2 \varphi)}} + h\right)\cos\varphi \cos\lambda \qquad Y = \left(\frac{a}{\sqrt{(1 - e^2 \sin^2 \varphi)}} + h\right)\cos\varphi \sin\lambda$$

$$Z = \left[\frac{a}{\sqrt{(1 - e^2 \sin^2 \varphi)}}(1 - e^2) + h\right]\sin\varphi$$

in cui **a** è il semiasse maggiore ed **e** l’eccentricità prima numerica dell’ellissoide di riferimento geodetico

Diversamente accade per il *passaggio inverso* dalle coordinate cartesiane ortogonali nello spazio X, Y, Z a quelle geografiche ellissoidiche φ, λ, h.

Mentre, per la determinazione della longitudine λ si utilizza, generalmente, la relazione

$$\lambda = \arctan\frac{Y}{X} \pm 180°$$

oppure la formula di Krakiwsky & Vanicek (1982) che la determina senza bisogno di definirne il segno e quindi il suo fuso, numericamente più stabile in vicinanza dei poli

$$\lambda = 2\arctan\frac{Y}{X + \sqrt{X^2 + Y^2}}$$

Krakiwsky & Vaniček (1982)

oppure la formula di Vermeille (2004) con una stabilità numerica maggiore rispetto alla precedente

$$\lambda = \frac{\pi}{2} - 2\arctan\frac{X}{\sqrt{X^2+Y^2}+Y} \quad \text{per } Y \geq 0$$

$$\lambda = \frac{\pi}{2} + 2\arctan\frac{X}{\sqrt{X^2+Y^2}-Y} \quad \text{per } Y < 0$$

Vermeille (2004)

e per la determinazione dell'altitudine h, una volta ottenuta la latitudine φ, si utilizza

$$h = \frac{\sqrt{X^2+Y^2}}{\cos\varphi} - N$$

oppure altre formule alternative come

$$h = \frac{Z}{\sin\varphi} - N(1-e^2)$$

Bencini (1968)

$$h = \sqrt{X^2+Y^2}\cos\varphi + Z\sin\varphi - a\sqrt{(1 - e^2\sin^2\varphi)}$$

Bowring (1985)

la determinazione della latitudine φ, invece, risulta più complicata in quanto nessuna relazione semplice la collega a X, Y, Z. Essa richiede tecniche indirette (o iterative) o metodi diretti (formule chiuse) di gran lunga più complessi di quelle indirette. In letteratura numerosi sono gli studi e le procedure proposte da vari autori (vedere tabella 1).

Simple Iteration

procedimento descritto in vari testi di geodesia come in Bomford, Guy (1899-1996) *Geodesy* (4ª ed. 1980, p. 679)

$$\tan\varphi_1 = \frac{Z + \dfrac{a}{\sqrt{(1 - e^2\sin^2\varphi_0)}}\, e^2\sin\varphi_0}{\sqrt{X^2+Y^2}}$$

il procedimento iterativo inizia con il valore di φ fornito dalla $\tan\varphi_0 \approx \dfrac{Z(1-e'^2)}{\sqrt{X^2+Y^2}}$

in cui **e'** è l'eccentricità seconda numerica dell'ellissoide di riferimento geodetico

Tabella 1 **Conversione cartesiane geocentriche ↔ geografiche ellissoidiche.** **Elenco alfabetico, non esaustivo, per autore**
Aleksic (1987) - Aleksic, I. "Transformacija geodetskih elipsoidnih koordinata φ, λ, h u pravougle prostorne (X,Y,Z)", *Proceedings of the Conference "Osnovni geodetski radovi i oprema* za *njihovo izvoaenje", Savez drustava geodetskih inzenjera i geometara Jugoslavije*,1987, pp. 723-730 (in serbo-croato)
Andreev (1966, 1975) - Andreev, M. "Preobrazovanie prjamougol'nikh prostranstvennikh koordinat v geodezicheskie", *Geodezija i kartografija*, 9, 1966, pp. 12-14 (in russo) - Andreev, M. "Formuli dlja vichislenija geodezicheskoj shiroti po prostranstvennim prjamougol 'nim koordinatam", *Geodezija i kartografija*, 5, 1975, pp. 26-28 (in russo)
Anufriev (1974) - Anufriev, O.I. (1974) "Prognozirovanie dvizhenija iskusstvennykh sputnikov Zemli", *IZVUZ, Geodezija i aerofotos'emka*, 1, 1974, pp. 45-49 (in russo)
Baird (1964) - Baird, R. W. "Cartesian Coordinate to Geodetic Coordinate Conversions", White Sands Missile Range Data Reduction Directorate, June 1, 1964
Balandin (1988) - Balandin, V.N. "Preobrazovanie prostranstvennikh prjamougol'nikh koordinat v geodezicheskie, *Geodezija i kartografija*, 1, 1988, pp. 42-43 (in russo)
Banasik & Ligas (2011) - Banasik, Piotr & Ligas, Marcin "Conversion between Cartesian and geodetic coordinates on a rotational ellipsoid by solving a system of nonlinear equations", *Geodesy And Cartography*, Polish Academy of Sciences, Vol. 60, No 2, 2011, pp. 145-159

Barbarella, Maurizio & Gatti, Marco “Note sulla trasformazione da sistema geocentrico a sistema ellissoidico”, *Bollettino di Geodesia e Scienze Affini*, No. 2, 1993, pp. 109-132

Barbee (1982)
- Barbee, Terrence W. 1982 “Geodetic latitude of a point in space”, *J. Spacecr. Rockets*, 19, 1982, pp. 378-380

Barrio R. & Riaguas A. “Comparison of algorithms for the transformation from geocentric to geodetic coordinates”, *Rev. Acad. Cienc. Zaragoza.*, 48, 1993, pp. 135-43

Bartelme & Meissl (1975)
- Bartelme, Norbert & Meissl, Peter (1934-1982) “Ein einfaches, rasches und numerisch stabiles Verfahren zur Bestimmung des kürzesten Abstandes eines Punktes von einem sphäroidischen Rotationsellipsoid”, *Allgemeine Vermessungs-Nachrichten*, 82, 1975, pp. 436-439

Bencini (1968)
- Bencini, Pietro (1921-1992) "Sul calcolo di trasformazione delle coordinate dal sistema geocentrico al sistema geografico", *Bollettino di Geodesia e Scienze Affini*, Vol. XXVII, No. 3, 1968, pp. 357-367

Benning (1974, 1987) [procedimento diretto]
- Benning, W. “Der kürzeste Abstand eines in rechtwinkligen Koordinaten gegebenen Außenpunktes vom Ellipsoid”, *Allgemeine Vermessungs-Nachrichten*, Vol. 81, No. 11, 1974, pp. 429-433
- Benning, W. “Iterative ellipsoidische Lotfußpunktberechnung“, *Allgemeine Vermessungs-Nachrichten*, Vol. 94, No. 7, 1987, pp. 256-260

Berger & Ricupito (1960)
- Berger, W.J. & Ricupito, J.R. “Geodetic latitude and altitude of a satellite”, *Journal of Advanced Robotic Systems*, Vol. 30, No. 9, 1960, pp. 901-902

Bopp & Krauss (1976)
- Bopp, H. & Krauss, H. “Der Kürzeste Abstand eines Punktes von einem Rotationsellipsoid”, *Allgemeine Vermessung-Nachrichten*, Vol. 83, No. 10, 1976, pp. 348-351

Borkowsky (1987, 1989) [procedimento diretto]
- Borkowski, Kazimierz M. “Transformation of Geocentric to Geodetic Coordinates without approximations”, Torun Radio Astronomy Observatory (Poland), *Astrophysics and Space Science*, 139, 1987, pp. 1-4
- Borkowski, Kazimierz M. “Correction”, *Astrophys. Space Sci.*, 146, 1988, p. 201
- Borkowski, Kazimierz M. “Accurate Algorithms to transform Geocentric to Geodetic Coordinates”, *Bulletin Géodésique*, No. 63, 1989, pp. 50-56

Bowring (1976, 1985) [procedimento diretto]
- Bowring, Bernard Russel (1925-2006) “Transformation from Spatial to Geographical Coordinates”, *Survey Review,* Vol. XXIII, No. 181, 1976, pp. 323-327
- Bowring, Bernard Russel (1925-2006) “*The accuracy of geodetic latitude and height equations”,* Survey Review*,* 28, 1985, pp. 202-206

Burtch, Robert “A comparison of methods used in rectangular to geodetic coordinate transformations”, *Proceedings of the American Congress on Surveying and Mapping (ACSM) Annual Conference and Technology Exhibition*, Orlando, April 21-26, 2006

Butkevich (1962, 1963, 1967, 1968, 1985)
- Butkevich, Adol’f Veniaminovich “Computation of the Geocentric Radius vector and latitude”, *IZVUZ, Geodezija i aerofotos'emka* 5, 1962 (in russo)
- Butkevich, Adol’f Veniaminovich “Perekhod ot prostranstvennykh prjamougol'nykh koordinat k geodezicheskim”, *Geodezija i kartografija*, 10, 1963, pp. 12-17 (in russo)
- Butkevich, Adol’f Veniaminovich “0 perekhode ot prostranstvennikh prjamougol'nikh koordinat k geodezicheskim”, *Geodezija i kartografija*, 6, 1967 (in russo)
- Butkevich, Adol’f Veniaminovich “K voprosu 0 perekhode ot prostranstvennikh prjamougol'nikh koordinat k geodezicheskim”, *Trudy Novosibirskogo instituta inzh. geodezii,aerofotos'emki i kartografii*, 22, 1968 (in russo)
- Butkevich, Adol’f Veniaminovich “Perekhod ot prostranstvennikh prjamougol' nikh koordinat k geodezicheskim bez priblizhenij”, *Geodezija, kartografija i aerofotos'emka*, Vol. 41, No. 41, 1985, pp. 3-6 (in russo)

Butkevich & Rad'o (1982)
- Butkevich, A.V. & Rad'o, T.V. “0 perekhode ot prostranstvennikh prjamougol'nikh koordinat k geodezicheskim bez priblizhenij”, *Geodezija i kartografija*, 5, 1982, pp. 16-17 (in russo)

Carlson (1980)
- Carlson, Neil A. “Fast geodetic coordinate transformations”, *Proceedings of AIAA Guidance and Control Conference*, *AIAA Conference Proceedings Series*, AIAA, Reston, 1980, pp. 328-337

Chen (1981)
- Chen, J.Y. “Geodetic datum and doppler positioning”, dissertation, Graz

Churchyard (1986)
- Churchyard, J.N. “Comment on ‘A new algorithm for the computation of the geodetic coordinates as a function of earth-centered earth-fixed coordinates”, *J. Guid. Control Dyn.*, 9, 1986, p. 511

Civicioglu, Pinar “Transforming geocentric cartesian coordinates to geodetic coordinates by using differential search algorithm”, *Computers & Geosciences*, Vol. 46, September 2012, pp. 229-247

Czarnecki (1972)
- Czarnecki, A. “Transformacja wsp6lrzednych prostokatnych x,y,z na wsp6lrzedne geodezyjne B,L i wysokosc elipsoidalna H”, *Geod. i kartogr.*, 21, No.3., 1972 (in polacco)

Czarnecki (1988)
- Czarnecki, K. “Conversion of rectangular x,y,z-coordinates into ellipsoidal (geodetic) h, φ, λ-coordinates. *Kart og plan*, 2, 1988, pp. 135-139

Crocetto (1993)
- Crocetto, Nicola “Point projection of the topographic surface onto the reference ellipsoid of revolution in geocentric Cartesian

coordinates", *Survey Review*, 32, 1993, pp. 233-238

Danielsen (1978)
- Danielsen, J. "Ein direkter Ubergang von kartesischen zu geografischen Koordinaten", *Allgemeine Vermessungs-Nachrichten*, 10, 1978, pp. 377-380

Daskalova (1969)
Daskalova, M. "Varhu transformatsijata na prostranstveni dekartovi koordinati v geodezicheski geografski koordinatie", *Izv. Tsentr. labor. geode Balg.* AN 10, 1969 (in bulgaro)
- Daskalova, M., Andreev, M. and Pejchev, S. "Varhu transformatsijata na prostranstvenite dekartovi koordinati v geodezichni geografski koordinati", 1973 (in bulgaro)

Deprit & Deprit-Bartholome (1975)
- Deprit, A. & Deprit-Bartholome, A. "Conversion from geocentric to geodetic coordinates", *Celestial Mechanics*, 12, 1975, pp. 489-493

Diaz-Toca & Marin & Necula (2019)
- Diaz-Toca, Gema M & Marin, Leandro & Necula, Ioana "Direct transformation from Cartesian into geodetic coordinates on a triaxial ellipsoid", 2019

Ecker (1967) [procedimento diretto]
- Ecker, E. "Die Normaenfällung auf das Ellipsoid", Österrich Zeitschrift für Vermessung und Photogrammetrie, 3, 1967, pp. 82-85

Eissfeller (1975)
- Eissfeller, B. "A Taylor Series Expansion for the Transformation Problem of Cartesian Baseline Components into Ellipsoidal Coordinate Differences", *Schriftenreihe, Universitaerer Studiengang Vermessungswesen, Universitaet der Bundeswehr Muenchen,* Heft 19, 1975, pp. 47-64

Featherstone, W.E. & Claessens, S.J. "Closed-Form Transformation between Geodetic and Ellipsoidal Coordinates", *Stud. Geophys. Geod.*, 52, 2008, pp. 1-18

Feltens (2008)
- Feltens, J. "Vector methods to compute azimuth, elevation, ellipsoidal normal, and the Cartesian (X, Y, Z) to geodetic (φ, λ, h) transformation", *Journa of Geodesy*, Vol. 82, 2008, pp. 493-504

Fok H.S. & Bâki Iz H. "A comparative analysis of the performance of iterative and noniterative solutions to the Cartesian to geodetic coordinate transformation", *Journal of Geospatial Engineering*, 5, 2003, pp. 61-74

Fotiou (1998) [procedimento diretto]
- Fotiou, A. "A pair of closed expressions to transform geocentric to geodetic coordinates", *Zeitschrift für Vermessungswesen*, 123, 1998, pp. 133-135

Frölich & Hansen (1976) [procedimento diretto]
- Frölich, H. & Hansen, H.H. "Zur Lotfußpunktberechnung bei rotationsellipsoidischer Bezugsfläche", *Allgemeine Vermessungs-Nachrichten*, Vol. 83, No. 5, 1976, 175-179

Fukushima (1999, 2006)
- Fukushima, Toshio "Fast transform from geocentric to geodetic coordinates", *Journal of Geodesy*, Vol. 73, No. 11, 1999, pp. 603-610
- Fukushima, Toshio "Transformation from Cartesian to geodetic coordinates accelerated by Halley's method", *Journal of Geodesy*, Vol. 79, No. 12, 2006. pp. 689-693

Gan'shin (1970)
- Gan'shin, V.N. "Perekhod ot prostranstvennikh prjamougol' nikh koordinat k geodezicheskim", *Geodezija i kartografija*, 1, 1970, pp. 17-19 (in russo)

Goad (1987)

Gerdan, George P. & Deakin, Rodney E. "Transforming Cartesian coordinates X,Y,Z to Geographical coordinates φ, λ, h", *The Australian Surveyor*, Vol. 44, No. 1, 1999, pp. 55-63

Gersten (1961) [procedimento diretto]
- Gersten, R.H. "Geodetic sub-latitude and altitude of a space vehicle", *Journal of Astronautical Sciences*, Vol. 3, No. 1, 1961, pp. 28-29

Grafarend (2001) [procedimento diretto]
- Grafarend, Erik Wilhelm (1939-2020) "Gauss surface normal coordinates in geometry and gravity space. Part 2a: topographic surface normal ellipsoidal coordinates: closed form transformation of "geocentric Cartesian" to "Gauss ellipsoidal coordinates (geodetic)", *Zeitschrift für Vermessungswesen*, 126, 2001, pp. 373-382

Grafarend & Lohse (1991)
- Grafarend, Erik Wilhelm (1939-2020). & Lohse, P. "The minimal distance mapping of the topographic surface onto the (reference) ellipsoid of revolution", *Manuscripta Geodaetica*, 16, 1991, pp. 92-110

Guo (2001)
- Guo, G. "A fast procedure for geodetic coordinate transformation", *Surveying and Mapping*, 11, 2001, pp. 12-13

Hmam (2018)
- Hmam, Hatem "Approximation solutions to the Cartesian to geodetic coordinate transformation problem", *International Global Navigation Satellite Systems Association IGNSS Symposium*, Colombo Theatres, Kensington Campus, UNSW Australia 7-9 February 2018

Heck (1987)
- Heck, B. "Rechenverfahren und Auswertemodelle der Landesvermessung", Wichmann, Karlsruhe, Germany (1987)

Hedgely (1976) [procedimento diretto]
- Hedgely, D.R. "An exact transformation from geocentric to geodetic coordinates for non-zero altitudes", *Technical Report R-458*, NASA, Maryland 1976

Hedman (1970)
- Hedman, Edward L. Jr. "A high-accuracy relationship between geocentric Cartesian coordinates and geodetic latitude and altitude", *Journal Spacecraft and Rockets*, Vol. **7**, No. 8 July-August, 1970, pp. 993-995

Heikkinen (1982) [procedimento diretto]
- Heikkinen, M. “Geschlossene Formeln zur Berechnung räumlicher geodätischer Koordinaten aus rechtwinkligen Koordinaten”, *Zeitschrift für Vermessungswesen*, Vol. 107, No. 5, 1982, pp. 207-211

Heindl (1997)
- Heindl, Gerhard “How to compute interval inclusions of geodetic coordinates from interval inclusions of Cartesian coordinates”, *Reliable Computing*, 3, 1997, pp. 421-435

Heiskanen & Moritz (1967)
Heiskanen, Weikko Alexsanteri (1895-1971) & Moritz, Helmut *Physical Geodesy* (1967)

Hekimoğlu (1995)
- Hekimoğlu, Serif “Generalized iterative solution for geodetic coordinates from Cartesian coordinates”, *Bollettino Geodesia e Scienze Affini*, 54, 1995, pp. 109-120

Hirvonen (1964)
- Hirvonen, Reino Antero (1908-1989) “Praktische Rechenformeln für die dreidimensionale Geodäsie. Zeitschrift für Vermessungswesen”, Vol. 89, No. 5, 1964, pp. 145-151

Hirvonen & Moritz (1963)
- Hirvonen, Reino Antero (1908-1989) & Moritz, Helmut “Practical Computation of Gravity at High Altitudes”, Report 27, Institute of Geodesy, Photogrammetry and Cartography, Ohio State University, Columbus 1963

Hsu (1992) [procedimento diretto]
- Hsu, D.Y. “Closed form solution for geodetic coordinates transformations”, *National Technical Meeting Proceedings*, The Institute of Navigation, Fairfax, VA, 1992, pp. 397-400

Izotov (1969)
- Izotov, Alexander A. (1907-2017) “Preobrazovanie prostranstvennikh prjamougol'nikh koordinat v geodezicheskie koordinati”, *Geodezija i kartografija* 5, 1969, pp.6-7

Jones (2002) [procedimento diretto]
- Jones, G.C. “New solutions for the geodetic coordinate transformation”, *Journal of Geodesy*, Vol. 76, No. 8, 2002, pp. 437-446

Kadaj (2021)
- Kadaj, Roman “Getchell’s method for conversion of Cartesian-geocentric to geodetic coordinates - Its properties and Newtonian alternative”, *Journal of Applied Geodesy*, Vol. 15, No. 1, 2021, pp. 47-60

Keeler & Nievergelt (1998)
- Keeler, Stephen P. & Nievergelt, Yves “Computing geodetic coordinates”, *SIAM Review*, 40, 1998, pp. 300-309

Kumi-Boateng, Bernard & Ziggah, Yao Yevenyo “Accuracy assessment of cartesian (x, y, z) to geodetic coordinates (φ, λ, h) transformation procedures in precise 3d coordinate transformation. A case study of Ghana geodetic reference network”, *Journal of Geosciences and Geomatics*, 2016, Vol. 4, No. 1, 1-7

Krakiwsky & Vaníček (1982) [procedimento diretto]
Krakiwsky, Edward J. & Vaníček, Petr *Geodesy. The Concepts* (1982), p. 324

Lapaine (1990) [procedimento diretto]
- Lapaine, Miljenko “A new direct solution of the transformation problem of Cartesian into ellipsoidal coordinates” in Rapp, Richard H. & Sansò, Fernando, *Determination of the Geoid. Present and Future* (1990), pp. 395-404

Laping (1962, 1964)
- Laping, K.A. “Computation of the Coordinates and the Height of a Point Using Measured Azimuths of Normal Sections and Elevation Angles in Two Fixed Points”, *IZVUZ, Geodezija i aerofotos'emka* 1, 1962 (in russo)
- Laping, K.A. “0 preobrazovanii prjamougol'nykh prostranstvennykh koordinat v geodezicheskie koordinaty”, *Geodezija i kartografija* 6, 1964, pp. 33-35 (in russo)

Laskowski Piotr “Is Newton’s iteration faster than simple iteration for transformation between geocentric and geodetic coordinates?”, *Bulletin Géodésique*, 65, 1991, pp. 14-17

Lee, Yong Chang “Evaluation of the Applicability of Solution Methods for 3D Conversion from Cartesian to Geodetic Coordinates”, *Journal of the Korean Society of Surveying, Geodesy, Photogrammetry and Cartography*, Vol. 32, No. 2, 2014, pp. 173-180

Lester (1922)
- Lester, Jones E. “Conversion of rectangular to geodetic co-ordinates”, *The Geographical Journal*, 60, 1922, p. 90

Levin (1988)
- Levin, Joshua Zev “A rational parametric approach to latitude, longitude, and altitude”, *Navigation*, 35, 1988, pp. 361-370

Lin & Wang (1995)
- Lin, K.C. & Wang, J. “Transformation from geocentric to geodetic coordinates using Newton’s iteration”, *Bulletin Géodésique,* Vol. 69, 1995, pp. 300-303

Long (1975)
- Long, S.A.T. “General-altitude transformations between geocentric and geodetic coordinates”, *Celestial Mechanics*, Vol. 12, No. 2, Sept. 1975, pp. 225-230

Lupash (1985)
- Lupash, Lawrence O. “A new algorithm for the computation of the geodetic coordinates as a function of Earth-centered Earth-fixed coordinates”, *J. Guid. Control Dyn.*, 8, 1985, pp. 787-789

Mayer (1978)
- Mayer, M. “Theoretische Untersuchen über die Genauigkeit der dreidimensionalen terrestrischen Punktbestimmung”, *German Geodetic Commission*, Reihe C, Hefte 244, Munich, 1978

Medvedev, Pavel Aleksandrovich & Bykov, Leonid Vasilevich & Bykov, Vasiliy Leonidovich & Novorodskaya, Marina Vladimirovna & Sherstneva, Svetlana Ivanovna “Methods for Transformation of Rectangular Spatial Coordinates to Geodetic Coordinates”, *International Journal of Engineering & Technology*, Vol. 7, No. 4.38, 2018, pp. 1179-1186

Morrison & Pines (1961)
- Morrison, John & Pines, Samuel “The reduction from geocentric to geodetic coordinates”, *Astronomical Journal*, 66, 1961, pp. 15-

16

Nautiyal (1988)
- Nautiyal, Atul “Algorithm to generate geodetic coordinates from Earth-centered Earth-fixed coordinates”, *Journal of Guidance, Control and Dynamics*, 11, 1988, pp. 281-283

Nievergelt & Keeler (2000)
- Nievergelt, Yves & Keeler, Stephen P. “Computing geodetic coordinates in space”, *Journal Spacecraft and Rockets*, 37, 2000, pp. 293-296

Olson (1988, 1996)
- Olson, Donald Karl II “Calculation of geodetic coordinates from Earth-centered Earth-fixed coordinates”, *Journal of Guidance, Control and Dynamics*, 11, 1988, pp. 188-190
- Olson, Donald Karl II “ Converting Earth-centered, Earth-fixed coordinates to geodetic coordinates”, *IEEE Transactions on Aerospace and Electronic Systems*, 32, 1996, pp. 473-476

Ozone (1985) [procedimento diretto]
- Ozone, Mohammed Id “Non-Iterative Solution of the φ Equation”, *Surveying and Mapping,* Vol. 45, No. 2, 1985, pp. 169-171
- Vincenty, T. “Comment and Discussion, Re: Non-Iterative Solution of the φ Equation, by Mohammed Id Ozone”, *Surveying and Mapping,* Vol. 45, No. 3, 1985, pp. 265-267

Panou, G. & Korakitis, R. & Delikaraoglou, D. “Cartesian to Geodetic Coordinates Conversion by an Iterative Geometrical Method”, Quod Erat Demonstrandum - In quest of the ultimate geodetic insight, Special issue for Professor Emeritus Athanasios Dermanis - School of Rural and Surveying Engineering, AUTh, 2018, pp. 274-281

Paul (1973) [procedimento diretto]
- Paul, M.K. “A Note on Computation of Geodetic Coordinates from Geocentric (Cartesian) Coordinates”, *Bulletin Géodésique,* No. 108, 1973, pp. 134-139

Pavlov (1967, 1968, 1970, 1971, 1973, 1975)
- Pavlov, K. “Sur la transformation de coordonnees rectangulaires speciales X, Y, Z en coordonnees geographiques φ, λ, h”, *Geod., kartograf., zemeustr.,* No.5, 1967 (in bulgaro)
- Pavlov, K. “Sur la transformation de coordonnees rectangulaires speciales X, Y, Z en coordonnees geographiques φ, λ, h pour les hauteurs inferieures a 10 km” *Schweizerische Zeitschrift für Vermessung, Photogrammetrie und Kulturtechnik*, 66, No. 12, 1968, pp. 381-388
- Pavlov, K. “Oshte vednazh za transformatsijata na pravoagalni prostranstveni koordinati x,y,z v geografski koordinati φ, λ, h”, Izv. Gl. upr. geode i kart., No.1, 1970 (in bulgaro)
- Pavlov, K. “Neiterativnite formuli za velichinite e^2NsinB i e^2NcosB pri transformatsijata na pravoagalni prostranstveni koordinati v geografski”, *Izv. Gl. upr. geode i kartogr.* 4, 1971 (in bulgaro)
- Pavlov, K. “Novi neiterativni formuli za transformatsija na pravoagalni prostranstveni koordinati v geografski koordinati”, Geod., kartograf., zemeustr., No.4, 1971 (in bulgaro)
- Pavlov, K. “Transformatsija na pravoagalni prostranstveni koordinati X, Y ,Z v geografski koordinati φ, λ, h”, Sofia 1973 (in bulgaro)
- Pavlov, K. “Opredeljane na geografskata shirina "B" s diferentsialni popravki pri malki visochini”, Izv. Gl. upr. geode i kartogr. 3, 1975 (in bulgaro)

Pejchev (1969)
- Pejchev, S. “Transformatsija na prostranstveni pravoagalni koordinati v geodezicheski”, *Izv. Centro labor. geode Balg. AN.,* 9, 1969

Penev (1973, 1978, 1980) [procedimento diretto]
- Penev, P. “Zatvoreni formuli za transformatsija na pravoagalni prostranstveni koordinati v geografski koordinati”, *Izv. Gl. upr. geode i kartogr.* 4, 1973 (in russo)
- Penev, P. “The transformation of rectangular coordinates into geographical by closed formulas”, *Geodesy Mapping and Photogrammetry*, 20, 1978, pp. 175-177
- Penev, P. “Transformatsija prjamougol'nikh koordinat v geograficheskie s primeneniem zamknutikh formul”, *IZVUZ, Geodezija i aerofotos'emka* 3, 1980, pp. 30-33 (in russo)

Pick (1967, 1985) [procedimento diretto]
- Pick, Miloš (1923-2007) “Transformation of the spatial rectangular coordinates into the geodetic coordinates”, *Bulletin Géodésique*, 83, 1967, pp. 21-26
- Pick, Miloš (1923-2007) “Transformation rechtwinkliger raumlicher Koordinaten in geodatische“, *Geofysikalni sbornik NCSAV*, 1966, Sv.14, 1967
- Pick, Miloš (1923-2007) “Closed formulas for the transformation of the Cartesian coordinate system into a system of geodetic coordinates”, *Stud. Geophys. et Geod.*, 29, 1985, pp. 112-119

Polevoj & Kim & Filippov (1970)
- Polevoj, V.A. & Kim, A.S. & Filippov, D.S. “0 perekhode ot geodezicheskikh koordinat k prostranstvennim prjamougol 'nim i obratno”, *Proceedings Geol., geofiz.,geodezija,* Nedra, Moskva 1970

Pollard (2002, 2005)
- Pollard, J. “Iterative vector methods for computing geodetic latitude and height from rectangular coordinates”, *Joiurnal of Geodesy*, 76, 2002, pp. 36-40
- Pollard, J. “A new approach to the iterative calculation of geodetic latitude and its calculation”, *Survey Review,* 38, 2005, pp. 117-123

Purcell & Cowan (1961)
- Purcell, E.W. & Cowan W.B. “Relating geodetic latitude and altitude to geocentric latitude and radius vector”, *Journal of Advanced Robotic Systems*, Vol. 31, No. 7, 1961, pp. 932-934

Rinner (1978)
- Rinner, Karl (1912-1991) “Geometrie mit Raumstrecken”, *Zeitschrift für Vermessungswesen*, Vol. 89, No. 3, 1978, pp. 91-105

Seemkooei, A.A. “Comparison of different algorithms to transform geocentric to geodetic coordinates”, *Survey Review*, 36, 2002, pp. 627-633

Shu, Chanfang & Li, Fei “An iterative algorithm to compute geodetic coordinates”, *Computers & Geosciences*, Vol. 36, 2010, pp. 1145-1149

Sjöberg (1999)
- Sjöberg, Lars E. “An efficient iterative solution to transform rectangular geocentric coordinates to geodetic coordinates”, *Zeitschrift für Vermessungswesen*, 124, 1999, pp. 295-297

Sofair (1985, 1997, 2000)
- Sofair, Isaac “A method for calculating exact geodetic latitude and altitude”, NSWC TR 85-85, April 1985, revised March 1993
- Sofair, Isaac “Improved method for calculating exact geodetic latitude and altitude”, *J. Guid. Control Dyn*., 20, 1997, pp. 824-826
- Sofair, Isaac “Improved method for calculating exact geodetic latitude and altitude revisited”, *J. Guid. Control Dyn*., 23, 2000, p. 369

Stoppini, Aurelio “Trasformazione diretta e inversa tra coordinate geografiche ellissoidche e coordinate cartesiane ellissocentriche”, *Bollettino di Geodesia e Scienze Affini*, Vol. LI, No. 1, 1992, pp. 51-64

Sudano (1997)
- Sudano, J.J. “An exact conversion from an Earth-centered coordinate system to latitude, longitude and altitude”, *Proceedings of the IEEE National Aerospace and Electronics Conference NAECON, Vol.2*, IEEE Aerospace & Electronics Systems Society, 1997 pp. 646-650

Sugai (1967) [procedimento diretto]
- Sugai, I. “Note on exact geodetic sub-latitude and geodetic altitude of a space vehicle”, *Journal of the Astronautical Sciences*, Vol. 14, No. 3, 1967, pp. 134-137

Sünkel (1976) [procedimento diretto]
- Sünkel, Hans “Ein nicht-iteratives Verfahren zur Transformation geodätischer Koordinaten”, *Österreichische Zeitschrift für Vermessungswesen*, Vol. 64, No. 1, 1999, pp. 29-33

Tomellieri (1970) [procedimento diretto]
- Tomellieri, Valentino (1926-1992) “L’esteroide tetracuspide retto-ellittico bi-alettato e le coordinate alti-geografiche ellissoidiche di un punto assegnato”, *Bollettino di Geodesia e Scienze Affini*, Vol. XXIX, No. 1, 1970, pp. 25-74 e No. 2, 1970, pp. 137-166

Toms (1995, 1996, 1998)
- Toms, Ralph M. “An efficient algorithm for geocentric to geodetic coordinate conversion”, *Proceedings on Standards for the Interoperability of Distributed Simulations, Vol.1*, Institute for Simulation and Training, Orlando, 1995, pp. 635-642
- Toms, Ralph M. “An improved algorithm for geocentric to geodetic coordinate conversion”, *Proceedings on Standards for the Interoperability of Distributed Simulations, Vol.2*, Institute for Simulation and Training, Orlando, 1996, pp. 1135-1144
- Toms, Ralph M. “New efficient procedures for geodetic coordinate transformations”, *Proceedings on Standards for the Interoperability of Distributed Simulations, Vol.3*, Institute, for Simulation and Training, Orlando, 1998, pp. 1024-1033

Torge (1975
Torge, Wolfgang *Geodäsie* (1975), *Geodesy* (1ª ed. inglese, 1980)

Unguendoli (1974)
- Unguendoli, Marco (1937-2003) "Relazioni fra coordinate cartesiane spaziali e coordinate geografiche (Inversione del Sistema)", *Bollettino della SIFET*, No. 1, 1974, pp. 39-42

Varao-Romao, M. Salomé “Transformacao de Coordenadas Cartesianas Tridimensionais em Geograficas por um processo directo”, *Revista do Instituto Geografico e Cadastral*, No. 7, Dezembro 1987, Publicacao Anual, pp. 87-94

Vermeille (2002, 2004, 2011) [procedimento diretto]
- Vermeille, Hugues “Direct transformation from geocentric to geodetic coordinates”, *Journal of Geodesy*, 76, 2002, pp. 451-454
- Vermeille, Hugues “Computing geodetic coordinates from geocentric coordinates” *Journal of Geodesy*, 78, 2004, pp. 94-95
- Vermeille, Hugues “An analytical method to transform geocentric into geodetic coordinates”, *Journal of Geodesy*, Vol. 85, 2011 pp. 105–117

Vincenty (1976, 1978, 1980)
- Vincenty, Thaddeus (1920-2002) “Ein Verfahren zur Bestimmung der geodätischen Höhe eines Punktes”, *Allgemeine Vermessungs-Nachrichten*, Vol. 83, No. 5, 1976, p. 179
- Vincenty, Thaddeus (1920-2002) “Vergleich zweier Verfahren zur Berechnung der geodätischen Breite und Höhe aus rechwinkligen Koordinaten”, *Allgemeine Vermessungs-Nachrichten*, Vol. 85, No. 7, 1978, pp. 269-270
- Vincenty, Thaddeus (1920-2002) “Zur räumlich-ellipsoidischen Koordinaten-Transformation”, *Zeitschrift für Vermessungswesen*, 105, 1980, pp. 519-521

Voll, Robert W. “Comparing the Accuracy and Efficiency of Algorithms for Converting Cartesian to Geodetic Coordinates”, Project Report submitted to the Faculty of the Virginia Polytechnic Institute and State University, 1990

Wei (1986) [procedimento diretto]
- Wei, Z. “Positioning with NAVSTAR, The Global Positioning System”, Department of Geodetic Science and Surveying, The 50 Ohio State University, Report No. 37@, Columbus, OH. August, 1986

Wu & Wang & Hu (2003)
- Wu, Yuanxin & Wang, Ping & Hu, Xiaoping “Algorithm of Earth-centered Earth-fixed coordinates to geodetic coordinates”, *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 39, No. 4, 2003, pp. 1457-1461

You (2000)
- You, Rey-Jer “Transformation of Cartesian to geodetic coordinates without iterations”, *Journal of Surveying Engineering-ASCE*, 126, 2000, pp. 1-7

Zeng (2013)
- Zeng, Huaien “Explicitly computing geodetic coordinates from Cartesian coordinates”, *Earth Planets Space*, Vol. 65, 2013, pp. 291-298

Zhang & Hsu & Wu & Li & Wang & Chai & Du (2005) [procedimento diretto]
- Zhang, C.D. & Hsu, H.T. & Wu, X.P. & Li, S.S. & Wang, Q.B. & Chai, H.Z. & Du L. “An alternative algebraic algorithm to transform Cartesian to geodetic coordinates”, *Journal of Geodesy*, Vol. 79, 2005, pp. 413-420

Zhu (1993, 1994)

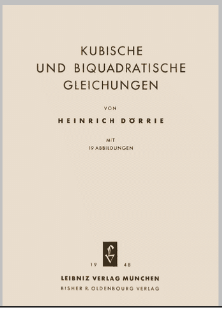
- Zhu, Jijie "Exact conversion of Earth-centered, Earth-fixed coordinates to geodetic coordinates", *J. Guid. Control Dyn*., 16, 1993, pp. 389-391
- Zhu, Jijie "Conversion of Earth-centered Earth-fixed coordinates to geodetic coordinates", *IEEE Transactions on Aerospace and Electronic Systems*, 30, 1994, pp. 957-961

Zlatanov (1972)

- Zlatanov, G. "Algoritam za transformatsija na pravoagalni ekvatorialnii koordinati (X,Y,Z) v geodezicheski (B,L,H) i preminavane kam nov referenten elipsoid", *Izv. Gl. upr. geod. i kartogr.,* No.4, 1972 (in bulgaro)

NOTE

- alcune forme dirette o chiuse (non iterative) mantengono tale proprietà solo per h=0 (punto giacente sulla superficie di riferimento ellissoidica);
- Vincenty (1985) afferma che la prima soluzione a questo problema è stata data da Dörrie, Heinrich (1873-1955) in *Kubische und biquadratische Gleichungen* (1948);



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