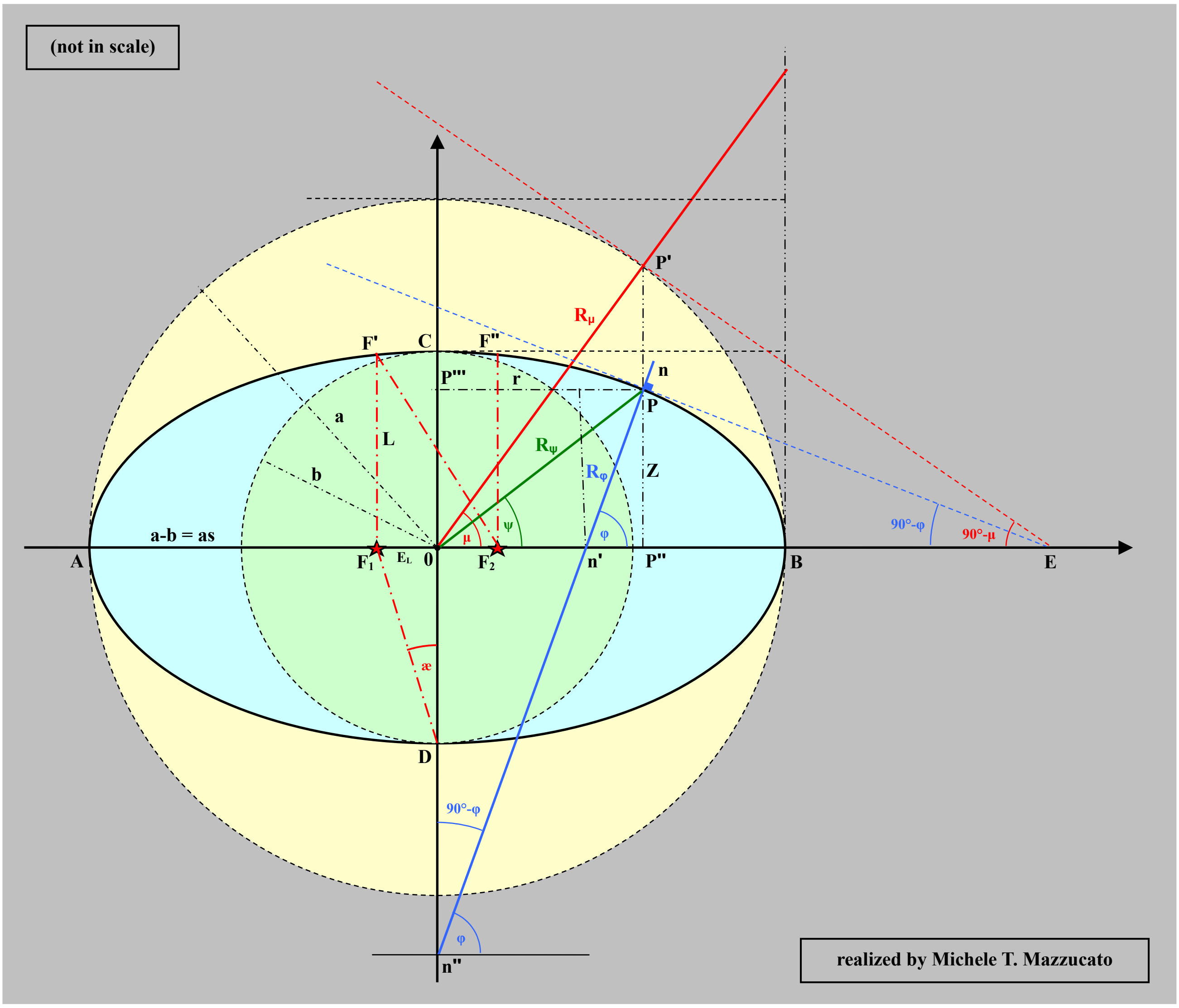
Geometrical relations on Meridian Section



known data	derived data
a equatoriale radius (6378388,000 m) [6378137,000 m] **b** polar radius (6367911,946 m) [6356752,314 m]	$e^2 = \frac{a^2 - b^2}{a^2}$ first eccentricity (0,006722670022) [0,006739496743] $s = \frac{a - b}{a}$ flattening (0,003367003367) [0,003352810665]
a equatoriale radius (6378388,000 m) [6378137,000 m] **s** flattening (0,003367003367) [0,003352810665]	$e^2 = 2s - s^2$ first eccentricity (0,006722670022) [0,006739496743] $b = a(1 - s) = a\sqrt{1 - e^2}$ polar radius (6367911,946 m) [6356752,314 m]

(values IRE24) [values WGS84]

Note: relations between e, e', s and E_L

$$(1-s)^2=(1-e^2) \qquad 1-s=\sqrt{1-e^2}=\frac{b}{a}=\frac{e}{e'} \qquad (1-e^2)(1+e'^2)=1 \qquad 1/s=\frac{e'}{e'-e} \qquad e=\frac{E_L}{a}$$

$$e^2=\frac{e'^2}{1+e'^2}=\frac{a^2-b^2}{a^2} \qquad e'^2=\frac{e^2}{1-e^2}=\frac{a^2-b^2}{b^2}$$

Note: canonical (or normal) equation of the ellipse in cartesian coordinates

$$\frac{X^2}{a^2}+\frac{Z^2}{b^2}=1 \quad \text{and} \quad b^2X^2+a^2Z^2=a^2b^2 \quad \text{equation in not canonical form}$$

and in polar coordinates

$$r_{\theta 1}=\frac{a(1-e^2)}{1-e\cos\theta}=\frac{L}{1-e\cos\theta} \ \text{origin on focus } F_1 \quad \text{and} \quad r_{\theta 2}=\frac{a(1-e^2)}{1+e\cos\theta}=\frac{L}{1+e\cos\theta} \ \text{origin on focus } F_2$$

$$\overline{F_1F'}+\overline{F_2F'}=2a=\text{constant}$$

equatorial radius (raggio equatoriale)

$a=\overline{0P''}+\overline{P''B}=\overline{F_1C}=\overline{F_1D}=\overline{F_2C}=\overline{F_2D}=\overline{0A}=\overline{0B}$	**(6 378 388,000 m)** **[6 378 137,000 m]**

polar radius (raggio polare)

$b=\overline{0C}=\overline{0D}=\overline{0n'}+\overline{n'P''}+\overline{P''0'}$	**(6 356 911,946 m)** **[6 356 752,314 m**

radius of curvature in the prime vertical (raggio di curvatura in primo verticale - gran normale)

$\overline{n''P}=N=\dfrac{a}{\sqrt{1-e^2\sin^2\varphi}}=\dfrac{r}{\cos\varphi}=N(1-e^2)+Ne^2=R_\varphi+Ne^2$	**(6 382 220,751 m)** **[6 381 953,457 m]**

3D cartesian coordinate (coordinate cartesiane 3D)

$X=\overline{0P''}=N\cos\varphi\cos\lambda$	**(5 784 256,365 m)** **[5 784 014,115 m]**
$Y=N\cos\varphi\sin\lambda$	**(0,000 m)** **[0,000 m]**
$Z=\overline{PP''}=R_\psi\sin\varphi=N(1-e^2)\sin\varphi$	**(2 679 110,365 m)** **[2 679 074,463 m]**

latitude and colatitude ellipsoidal (latitudine e colatitudine ellissoidica)

$\tan\varphi=\dfrac{a^2}{b^2}\tan\psi=\dfrac{a}{b}\tan\mu=\dfrac{a^2Z}{b^2r}=\dfrac{Z}{r(1-e^2)}=\dfrac{Z}{N(1-e^2)\cos\varphi}$ $\chi_\varphi=90°-\varphi$	**(25°)** **(65°)** **[25°]** **[65°]**

latitude and colatitude ellipsocentric (latitudine e colatitudine ellissocentrica)

$\tan\psi=\dfrac{\overline{n'P}\sin(180°-\varphi)}{\overline{0n'}-\overline{n'P}\cos(180°-\varphi)}=$ **Law of Cosines** $=\dfrac{N(1-e^2)\sin(180°-\varphi)}{Ne^2\cos\varphi-N(1-e^2)\cos(180°-\varphi)}$ $\tan\psi=\dfrac{Z}{X}=\dfrac{b^2}{a^2}\tan\varphi=\dfrac{b}{a}\tan\mu \qquad \chi_\psi=90°-\psi$	**(24,852290°)** **(65,147710°)** **[24,852913°]** **[65,147087°]**

Note: $\cos^2\psi + \sin^2\psi = 1$

latitude and colatitude parametric (latitudine e colatitudine parametrica)

$\tan\mu = \sqrt{1 - e^2}\tan\varphi = \frac{a}{b}\tan\psi$ $\chi_\mu = 90° - \mu$	(24,926065°) (65,073935°) [24,926377°] [65,073623°]

Note on latitudes: in absolute value Ψ < μ < φ, except at the poles and at the equator where they coincide.

parallel radius (raggio del parallelo)

$\overline{PP'''} = r = \frac{a}{\sqrt{1 - e^2\sin^2\varphi}}\cos\varphi = N\cos\varphi = a\cos\mu = R_\psi\cos\psi$	**(5 784 256,365 m)** **[5 784 014,115 m]**

ellipsoidal radius (raggio ellissoidico)

$\overline{n'P} = R_\varphi = N(1 - e^2) = \frac{Z}{\text{sen}\varphi}$	**(6 339 315,187 m)** **[6 339 230,236 m]**

ellipsocentric radius (raggio ellissocentrico)

$\overline{0P} = R_\psi = N\cos\varphi\sqrt{1 + (1 - e^2)^2\tan^2\varphi} = \frac{\sqrt{1 - e^2(2 - e^2)\sin^2\varphi}}{\sqrt{1 - e^2\sin^2\varphi}} = \sqrt{\overline{0P''} + \overline{PP''}} = \sqrt{X^2 + Z^2}$ $\overline{0P} = R_\psi = R_\psi{}^2 + (Ne^2\cos\varphi)^2 - 2R_\varphi(Ne^2\cos\varphi)\cos(180° - \varphi) =$ **Law of Cosines** $= [N(1 - e^2)]^2 + (Ne^2\cos\varphi)^2 - 2N(1 - e^2)(Ne^2\cos\varphi)\cos(180° - \varphi)$ $\overline{0P} = R_\psi = \frac{ab}{\sqrt{(b\cos\psi)^2 + (a\sin\psi)^2}} = \frac{b}{\sqrt{1 - (e\cos\psi)^2}}$	**(6 374 578,734 m)** **[6 374 343,830 m]**

Note:

$Z = mX$ equation of the straight line passing through the origin of the ellipse with $m = \tan\psi$ slope of the straight line

$b^2X^2 + a^2Z^2 = a^2b^2$ equation in not canonical form of the ellipse

$$b^2X^2 + a^2(mX)^2 - a^2b^2 = b^2X^2 + a^2m^2X^2 - a^2b^2 = 0$$

$$X = \sqrt{\frac{a^2b^2}{b^2 + a^2m^2}} \quad \text{and} \quad Z = mX \qquad R_\psi = \sqrt{X^2 + Z^2}$$

parametric radius (raggio parametrico)

$\overline{0P'} = R_\mu = a$	**(6 378 388,000 m)** **[6 378 137,000 m]**

angular eccentricity (eccentricità angolare)

$æ = \arccos\frac{b}{a} = 2\arctan\sqrt{\frac{a - b}{a + b}} = \text{arcsen}\, e$	**(4,703069°)** **[4,693141°]**

linear eccentricity (eccentricità lineare)

$\overline{0F_1} = \overline{0F_2} = E_L = \sqrt{a^2 - b^2} = ae = a\sin æ$	(522 976,087 m) [521 854,008 m]
$\overline{PP''} = Z = \sqrt{\overline{n'P}^2 - \overline{OP''}^2} = R_\varphi \sin\varphi = \sqrt{\overline{0P}^2 - \overline{OP''}^2} = R_\psi \sin\psi = b\sin\mu$	(2 679 110,365 m) [2 679 074,463 m]
$\overline{P'P''} = \sqrt{\overline{0P'}^2 - \overline{0P''}^2} = \sqrt{a^2 - X^2} = a\sin\mu$	(2 688 161,413 m) [2 688 087,110 m]
$\overline{PP'} = \overline{P'P''} - \overline{PP''} = \sqrt{a^2 - X^2} - Z$	(9 051,049 m) [9 012,647 m]
$\overline{n'n''} = \overline{n''P} - \overline{n'P} = N - R_\varphi = Ne^2$	(42 905,564 m) [42 723,222 m]
$\overline{P''B} = \overline{0B} - \overline{0P''} = a - r$	(594 131,635 m) [594 122,885 m]
$\overline{0P''} = Ne^2\cos\varphi + N(1 - e^2)\cos\varphi = a\cos\mu$	(5 784 256,365 m) [5 784 014,115 m]
$\overline{n'P''} = R_\varphi \cos\varphi = N(1 - e^2)\cos\varphi$	(5 745 370,718 m) [5 745 293,726 m]
$\overline{0n'} = \overline{0P''} - \overline{n'P''} = Ne^2\cos\varphi$	(38 885,647 m) [38 720,388 m]
$\overline{P''0'} = \overline{00'} - \overline{0P''} = b - r$	(572 655,581 m) [572 738,200 m]
$\overline{0n''} = Ne^2\sin\varphi$	(18 132,675 m) [18 055,614 m]
$\overline{P'''n''} = N\sin\varphi$	(2 697 243,040 m) [2 697 130,077 m]
$\overline{PE} = \dfrac{\overline{PP''}}{\sin(90° - \varphi)} = \dfrac{\overline{PP''}}{\sin\chi_\varphi} = \dfrac{\overline{PP''}}{\cos\varphi} = \overline{n'E}\sin\varphi$	(2 956 071,219 m) [2 956 031,606 m]
$\overline{P'E} = \dfrac{\overline{P'P''}}{\sin(90° - \mu)} = \dfrac{\overline{P'P''}}{\sin\chi_\mu} = \dfrac{\overline{P'P''}}{\cos\mu} = \overline{0E}\sin\mu = a\tan\mu$	(2 964 276,723 m) [2 964 202,285 m]
$\overline{P''E} = \overline{P'P''}\tan\mu = \overline{P'E}\sin\mu = \overline{P'E}\cos\chi_\mu = \overline{0E} - r = \overline{PE}\tan\varphi = \overline{PE}\sin\varphi = \overline{PE}\cos$	(1 249 289,680 m) [1 249 272,939 m]
$\overline{n'E} = \dfrac{R_\varphi}{\cos\varphi}$	(6 994 660,398 m) [6 994 566,665 m]

$\overline{0E} = \dfrac{a}{\cos\mu}$	**(7 033 546,045 m)** **[7 033 287,054 m]**

focal distance (distanza focale)

$\overline{F_1F_2} = 2E_L = 2\sqrt{a^2 - b^2}$	**(1 045 952,174 m)** **[1 043 708,017 m]**

straight semilated (semilato retto)

$\overline{F_1F'} = \overline{F_2F''} = L = \dfrac{b^2}{a} = a(1 - e^2)$	**(6 335 508,202 m)** **[6 335 439,327 m]**

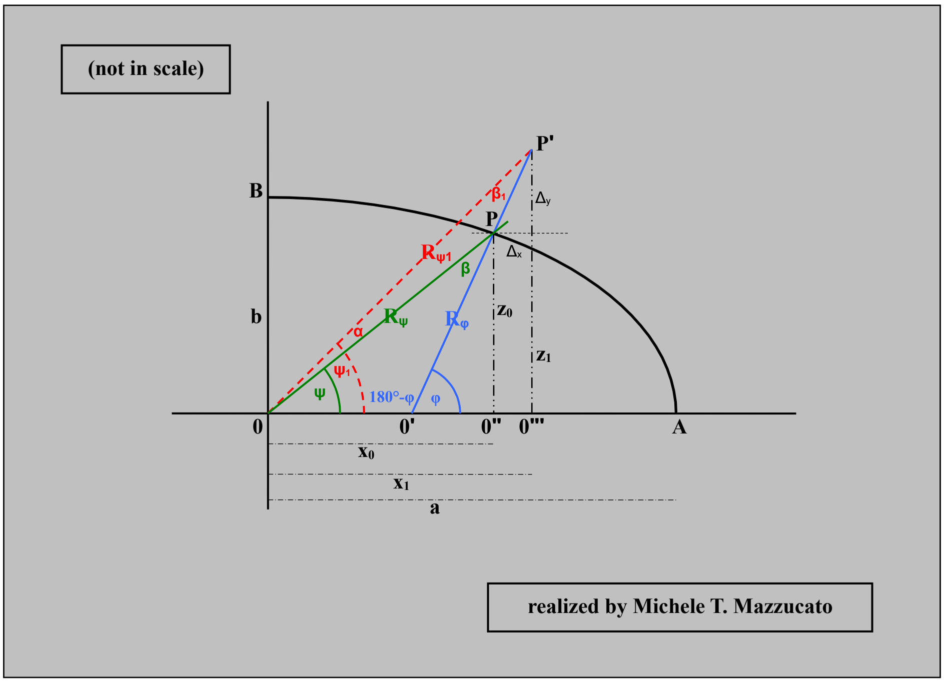
equatorial diameter (diametro equatoriale)

$\overline{AB} = \overline{F_1F'} + \overline{F_2F'} = 2a$	**(12 756 776,000 m)** **[12 756 274,000 m]**

polar diameter (diametro polare)

$\overline{CD} = 2b$	**(12 713 823,892 m)** **[12 713 504,628 m]**

$\overline{F_2F'} = \sqrt{\overline{F_1F'}^2 + 2E_L{}^2}$	**(6 421 267,798 m)** **[6 420 834,673 m]**



$\overline{00''} = X_0 = \overline{00'} + \overline{0'0''} = N\cos\varphi$	**(5 784 256,365 m)** **[5 784 014,115 m]**

$\overline{00'''} = X_1 = \overline{00'} + \overline{0'0''} + \overline{0''0'''} = (N + h)\cos\varphi$	**(5 784 703,175 m)** **[5 784 460,924 m]**

$\Delta_x = \overline{0''0'''} = \overline{0''A} - \overline{0'''A} = X_1 - X_0$	**(446,810 m)** **[446,809 m]**

$\overline{0''A} = a - X_0$	(594 131,635 m) [594 122,885 m]

$\overline{0''A} = a - X_1$	(593 684,825 m) [593 676,076 m]

$\overline{0'0''} = N(1-e^2)\cos\varphi$	(5 745 370,718 m) [5 745 293,727 m]

$\overline{00'} = \overline{00''} - \overline{0'0''} = X_0 - \overline{0'0''} = N\cos\varphi - N(1-e^2)\cos\varphi = Ne^2\cos\varphi$	(38 885,647 m) [38 720,388 m]

$\overline{P0''} = Z_0 = N(1-e^2)\sin\varphi$	(2 679 110,365 m) [2 679 074,463 m]

$\overline{P'0'''} = Z_1 = [N(1-e^2)+h]\sin\varphi$	(2 679 318,716 m) [2 679 282,814 m]

$\Delta_y = Z_1 - Z_0$	(208,351 m) [208,351 m]

$\overline{0'P} = R_\varphi = N(1-e^2)$	(6 339 315,187 m) [6 339 230,236 m]

$\overline{0P} = R_\psi = \sqrt{X_0{}^2 + Z_0{}^2}$ $\overline{0P} = R_\psi = \sqrt{\overline{00'}^2 + \overline{0'P}^2 - 2\,\overline{00'}\,\overline{0'P}\cos(180° - \varphi)} =$ **Law of Cosines** $= \sqrt{[Ne^2\cos\varphi]^2 + [N(1-e^2)]^2 - 2[Ne^2\cos\varphi][N(1-e^2)]\cos(180° - \varphi)}$ $\overline{0P} = R_\psi = \frac{\overline{0'P}\sin(180° - \varphi)}{\sin\psi} = \frac{\overline{00'}\sin(180° - \varphi)}{\sin\beta}$ **Law of Sines**	(6 374 578,734 m) [6 374 343,830 m]

Note: spherical coordinates

$$X = R_\psi \cos\psi\cos\lambda$$
$$Y = R_\psi \cos\psi\sin\lambda$$
$$Z = R_\psi \sin\psi$$

$$R_\psi = \sqrt{X^2 + Y^2 + Z^2} \qquad R_\psi = \frac{Z}{\sin\psi} = \frac{Y}{\cos\psi\sin\lambda} = \frac{X}{\cos\psi\cos\lambda} \qquad R_\psi = \frac{\sqrt{X^2+Y^2}}{\cos\psi}$$

$$\tan\psi = \frac{Z}{\sqrt{X^2+Y^2}} \qquad \cos\psi = \frac{\sqrt{X^2+Y^2}}{R\psi} \qquad \tan\lambda = \frac{Y}{X} \pm 180°$$

$\overline{0'P'} = N(1-e^2) + h$	(6 339 808,187 m) [6 339723,236 m]

$\overline{0P'} = R_{\psi_1} = \sqrt{X_1^2 + Z_1^2} = \sqrt{\overline{00'}^2 + \overline{0'P'}^2 - 2\,\overline{00'}\,\overline{0'P'}\cos(180° - \varphi)} =$ $= \sqrt{[Ne^2 \cos\varphi]^2 + [N(1-e^2)+h]^2 - 2[Ne^2\cos\varphi][N(1-e^2)+h]\cos(180° - \varphi)}$ **Law of Cosines** $\overline{0P'} = R_{\psi_1} = \frac{\overline{0'P'}\sin(180° - \varphi)}{\sin\psi_1} = \frac{\overline{00'}\sin(180° - \varphi)}{\sin\beta_1}$ **Law of Sines**	**(6 375 071,733 m)** **[6 374 836,828 m]**

$a = \overline{00'} + \overline{0'0''} + \overline{0''0'''} + \overline{0'''A}$	**(6 378 388,000 m)** **[6 378 137,000 m]**

$\overline{PP'} = h = R_{\psi_1} - R_{\psi} = \sqrt{\Delta_x^2 + \Delta_y^2}$	**(493,000 m)** **[493,000 m]**

$180° - \varphi$	**(155°)** **[155°]**

$\tan\psi = \frac{Z_0}{X_0}$ $\cos\psi = \frac{\overline{00'}^2 + \overline{0P}^2 - \overline{0'P}^2}{2\,\overline{00'}\,\overline{0P}} = \frac{\overline{00'}^2 + R_\psi^2 - R_\varphi^2}{2\,\overline{00'}\,R_\psi} =$ **Law of Cosines** $= \frac{[N\cos\varphi - N(1-e^2)\cos\varphi]^2 + R_\psi^2 - [N(1-e^2)]^2}{2[N\cos\varphi - N(1-e^2)\cos\varphi]R_\psi}$ $\sin\psi = \frac{\overline{0'P}\sin(180° - \varphi)}{\overline{0P}} = \frac{R\varphi\sin(180° - \varphi)}{R_\psi}$ **Law of Sines**	**(24,852290°)** **[24,852913°]**

$\tan\psi_1 = \frac{Z_1}{X_1}$ $\cos\psi_1 = \frac{\overline{00'}^2 + \overline{0P'}^2 - \overline{0'P'}^2}{2\,\overline{00'}\,\overline{0P'}} = \frac{\overline{00'}^2 + R_{\psi_1}^2 - (R_\varphi + h)^2}{2\,\overline{00'}\,R_{\psi_1}} =$ **Law of Cosines** $= \frac{[N\cos\varphi - N(1-e^2)\cos\varphi]^2 + R_{\psi_1}^2 - [N(1-e^2)+h]^2}{2[N\cos\varphi - N(1-e^2)\cos\varphi]R_{\psi_1}}$ $\sin\psi_1 = \frac{\overline{0'P'}\sin(180° - \varphi)}{\overline{0P'}} = \frac{(R\varphi + h)\sin(180° - \varphi)}{R_{\psi_1}}$ **Law of Sines**	**(24,852302°)** **[24,852924°]**

$\beta = 180° - [\psi + (180° - \varphi)]$ $\sin\beta = \frac{\overline{00'}\sin(180° - \varphi)}{\overline{0P}} = \frac{[N\cos\varphi - N(1-e^2)\cos\varphi]\sin(180° - \varphi)}{R_\psi}$ **Law of Sines**	**(0,147710°)** **[0,147087°]**

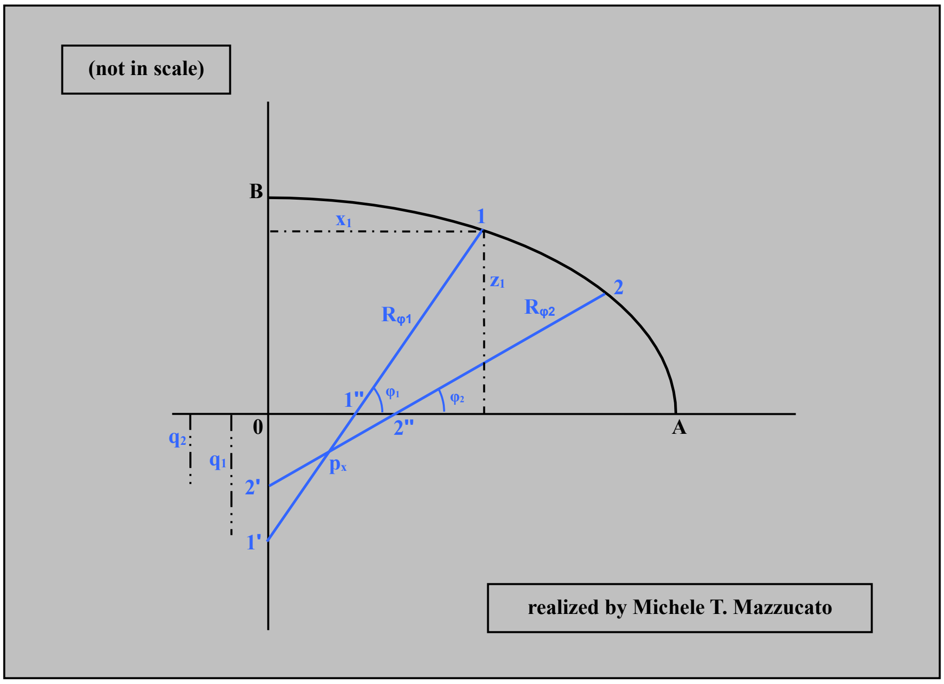
$\beta_1 = 180° - [\psi_1 + (180° - \varphi)]$ $\sin\beta_1 = \frac{\overline{00'}\sin(180° - \varphi)}{\overline{0P'}} = \frac{[N\cos\varphi - N(1-e^2)\cos\varphi]\sin(180° - \varphi)}{R_{\psi_1}}$ **Law of Sines**	**(0,147698°)** **[0,147076°]**

$\psi + (180° - \varphi) + \beta = 180°$	**(180°)** **[180°]**

$\psi_1 + (180° - \varphi) + \beta_1 = 180°$	**(180°)** **[180°]**

$\alpha = \psi_1 - \psi \qquad \sin\alpha = \frac{\overline{PP'}\sin\beta_1}{\overline{0P}} = \frac{h\sin\beta_1}{R_\psi}$ **Law of Sines**	**(0,000011423°)** **[0,000011375°]**

$\tan\varphi = \frac{Z_0}{R\varphi\cos\varphi} = \frac{Z_1}{R\varphi\cos\varphi + \Delta_x}$	**(25°)** **[25°]**



Note: equation of a straight line (equazione della retta)

equation of a straight line [implicit form] (equazione della retta in forma implicita)

$$ax + by + c = 0$$

equation of a straight line [explicit form] (equazione della retta in forma esplicita)

$$y = mx + q$$

where (con)

$m = -\dfrac{a}{b}$ and $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ angular coefficient o gradient of the straight line (coefficiente angolare della retta)

$q = -\dfrac{c}{b}$ and $q = \dfrac{x_2 y_1 - x_1 y_2}{x_2 - x_1}$ ordinate at the origin of the straight line (ordinata all'origine della retta)

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ cartesian distance between two points (distanza tra due punti)

equation of the straight line from explicit to implicit form (equazione della retta da forma esplicita a forma implicita)

$$y = mx + q \Rightarrow -mx + y - q = 0$$

φ_1 = 23°

λ_1 = 0°

$m_1 = \tan\varphi_1$ = 0,424474816

$q_1 = N_1 e^2 \sin\varphi_1$ = - 16763,076 m

$x_1 = N_1 \cos\varphi_1 \cos\lambda_1$ = 5874352,472 m

$y_1 = m_1 x_1 + q_1 \quad y_1 = Z_1 = R_{\varphi_1} \sin\varphi_1 = N_1 (1 - e^2) \sin\varphi_1$ = 2476751,610 m

point 1" coordinates on the X axis (coordinate punto 1" sull'asse delle X)

$x_{1''} = \overline{01''} = N_1 e^2 \cos\varphi_1 = N_1 \cos\varphi_1 \cos\lambda_1 - N_1 (1 - e^2) \cos\varphi_1$ = 39491,333 m

$y_{1''}$ = 0 m

$R_{\varphi_1} = d_{11''} = \sqrt{(x_1 - x_{1''})^2 + (y_1 - y_{1''})^2}$ = 6338761,949 m

$$q_1 = \frac{x_1 y_{1''} - x_{1''} y_1}{x_1 - x_{1''}} = -16763{,}076 \text{ m}$$

$$y_1 = m_1 x_1 + q_1 \Rightarrow -m_1 x_1 + y_1 - q_1 = 0$$

$\varphi_2 = 12°$

$\lambda_2 = 0°$

$m_2 = \tan\varphi_2 = 0{,}212556562$

$q_2 = N_2 e^2 \sin\varphi_2 = -8916{,}507 \text{ m}$

$x_2 = N_2 \cos\varphi_2 \cos\lambda_2 = 6239911{,}652 \text{ m}$

$y_2 = m_2 x_2 + q_2 \quad y_2 = Z_2 = R_{\varphi_2} \sin\varphi_2 = N_2(1 - e^2)\sin\varphi_2 = 1317417{,}659 \text{ m}$

point 2'' coordinates on the X axis (coordinate punto 2'' sull'asse delle X)

$x_{2''} = \overline{02''} = N_2 e^2 \cos\varphi_2 = N_2 \cos\varphi_2 \cos\lambda_2 - N_2(1 - e^2)\cos\varphi_2 = 41948{,}867 \text{ m}$

$y_{2''} = 0 \text{ m}$

$R_{\varphi_2} = d_{22''} = \sqrt{(x_2 - x_{2''})^2 + (y_2 - y_{2''})^2} = 6336428{,}961 \text{m}$

$$q_2 = \frac{x_2 y_{2''} - x_{2''} y_2}{x_2 - x_{2''}} = -8916{,}507 \text{ m}$$

$$y_2 = m_2 x_2 + q_2 \Rightarrow -m_2 x_2 + y_2 - q_2 = 0$$

Note: Cramer's rule for linear systems of 2 equations in 2 variables
(Metodo di Cramer per sistemi lineari di 2 equazioni in 2 incognite)

$$\begin{cases} a_1 x + b_1 y = c_1 \\ a_2 x + b_2 y = c_2 \end{cases}$$

matrix of coefficients (matrice dei coefficienti)

$$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$$

determinant of the matrix 2x2 (determinante della matrice 2x2)

$$D = (a_1 \cdot b_2) - (a_2 \cdot b_1)$$

if D = 0 indeterminate or impossible system, do not proceed (se D=0 sistema indeterminato o impossibile, non si procedere)
if D ≠ 0 system determined, proceed (se D≠0 sistema determinato, si procede)

determinant of the variable x (determinante dell'incognita x)

$$D_x = \begin{bmatrix} c_1 & b_1 \\ c_2 & b_2 \end{bmatrix} = (c_1 \cdot b_2) - (c_2 \cdot b_1)$$

determinant of the variable y (determinante dell'incognita y)

$$D_y = \begin{bmatrix} a_1 & c_1 \\ a_2 & c_2 \end{bmatrix} = (a_1 \cdot c_2) - (a_2 \cdot c_1)$$

2x2 system solutions (soluzioni del sistema 2x2)

$$x = \frac{D_x}{D} \quad y = \frac{D_y}{D}$$

[from Gabriel Cramer (1704-1752) in Introduction a l'analyse des courbes algebriques (1750) in which he introduces Cramer's rule for solving a system of linear equations. Colin Maclaurin (1698-1746) in Treatise of algebra (1748) also dealt with it and provided its proof for linear systems with two and three unknowns.]

[da Gabriel Cramer (1704-1752) in *Introduction a l'analyse des courbes algebriques* (1750) in cui introduce la *regola di Cramer* per la risoluzione di un sistema di equazioni lineari. Anche Colin Maclaurin (1698-1746) in *Treatise of algebra* (1748) ne trattò e ne fornì la dimostrazione per sistemi lineari con due e tre incognite.]

point of intersection between two straight lines (*elimination of variables metod*)
(punto di intersezione tra due rette, *metodo di sostituzione*)

$$\begin{cases} -m_1x_1 + y_1 = q_1 \\ -m_2x_2 + y_2 = q_2 \end{cases}$$

$$\begin{cases} -0{,}424474816x_1 + y_1 = -16763{,}076 \\ -0{,}212556562x_2 + y_2 = -8916{,}507 \end{cases}$$

$$\begin{cases} y = 0{,}424474816x - 16763{,}076 \\ -0{,}212556562x + y = -8916{,}507 \end{cases}$$

$$\begin{cases} y = 0{,}424474816x - 16763{,}076 \\ -0{,}212556562x + (0{,}424474816x - 16763{,}076) = -8916{,}507 \end{cases}$$

$$\begin{cases} y = 0{,}424474816x - 16763{,}076 \\ 0{,}211918254x - 16763{,}076 = -8916{,}507 \end{cases}$$

$$\begin{cases} y = 0{,}424474816x - 16763{,}076 \\ 0{,}211918254x = 7846{,}569 \end{cases}$$

$$\begin{cases} y = 0{,}424474816x - 16763{,}076 \\ x = \frac{7846{,}569}{0{,}211918254} = 37026{,}395 \end{cases}$$

$$\begin{cases} y_{p_x} = 0{,}424474816 \cdot 37026{,}395 - 16763{,}076 = -1046{,}304 \\ x_{p_x} = 37026{,}396 \end{cases}$$

point of intersection between two straight lines (*Cramer's rule*) for verification
(punto di intersezione tra due rette, *metodo di Cramer* per verifica)

$$\begin{cases} -m_1x_1 + y_1 = q_1 \\ -m_2x_2 + y_2 = q_2 \end{cases}$$

$$\begin{cases} -0{,}424474816x_1 + y_1 = -16763{,}076 \\ -0{,}212556562x_2 + y_2 = -8916{,}507 \end{cases}$$

$$\begin{bmatrix} -m_1 & 1 \\ -m_2 & 1 \end{bmatrix}$$

determinant of the matrix 2x2 (determinante della matrice 2x2)

$$D = (-m_1 \cdot 1) - (-m_2 \cdot 1) = -0{,}211918254$$

if D ≠ 0 system determined, proceed (se D≠0 sistema determinato, si procede)

determinant of the variable x (determinante dell'incognita x)

$$D_x = \begin{bmatrix} -q_1 & 1 \\ -q_2 & 1 \end{bmatrix} = (-q_1 \cdot 1) - (-q_2 \cdot 1) = -7846{,}569$$

determinant of the variable y (determinante dell'incognita y)

$$D_y = \begin{bmatrix} -m_1 & -q_1 \\ -m_2 & -q_2 \end{bmatrix} = (-m_1 \cdot -q_2) - (-m_2 \cdot -q_1) = 221{,}731$$

2x2 system solutions (soluzioni del sistema 2x2)

$$y_{p_x} = \frac{D_y}{D} = -1046{,}304 \qquad x_{p_x} = \frac{D_x}{D} = 37026{,}395$$